TMA 401/MAN 670 Functional Analysis 2001/2002
Peter Kumlin
Matematiska institutionen CTH& GU

Problem 3

hand in at latest May 15

1. Calculate the Green's functions for the boundary value problem

$$\begin{cases} u^{(4)}(x) = f(x) \\ u(0) = u''(0) = u(1) = u''(1) = 0 \end{cases}$$

2. Show that the problem

$$\begin{cases} u''(x) + u(x) = e^{ix} + \frac{1}{2}\text{Re } u(x), \ 0 \le x \le \pi/2 \\ u'(0) = u'(\pi/2) = 0, \ u \in C^2([0, \pi/2]) \end{cases}$$

has a unique solution.

3. The operator A on $L^2([0,1])$ is defined by

$$(Af)(x) = \int_0^x f(y)\, dy, \quad 0 \le x \le 1.$$

Find A^*.

4. Suppose S is a closed convex subset of a Hilbert space H and let P_S denote the orthogonal projection onto S, i.e. for any $x \in H$, $P_S(x)$ denotes the point in S, which is nearest to x. Prove that

$$\|P_S(x) - P_S(y)\| \le \|x - y\| \text{ for all } x, y \in H.$$

5. Set

$$(Ax)(t) = \int_{-\infty}^{\infty} \frac{x(s)}{1 + (t-s)^2}\, ds, \quad x \in L^2(\mathbf{R}).$$

Prove that A defines a linear bounded and self-adjoint operator on $L^2(\mathbf{R})$. Finally prove that A is not a compact operator.