Matematik, CTH & GU

Tentamen i Funktionalanalys TMA401/MAN670

Hjälpmedel: Inga (inte ens räknedosa).

Personuppgifter: Namn, personnummer, linje, antagningsår.

Inlämning ska ske i uppgifternas ordning; v.g. sidnumrera!

Teoriuppgifter: 4,5,6.

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Datum: 2002–06–01
Skrivtid: fm (5 timmar)
Lokal: maskin

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1. Let A be the linear mapping on $L^2([0,1])$ defined by

$$Af(x) = \int_0^1 (x-y)f(y)\,dy, \quad 0 \le x \le 1.$$

Calculate

(a) A^*A

(b) $\|A\|$.

(2+2p)

2. Prove the existence and uniqueness of a solution to the following boundary value problem

$$\begin{cases} -u''(x) = 2 + \dfrac{1}{1+u^2(x)}, & 0 \le x \le 1 \\ u(0) = u(1) = 0, & u \in C^2([0,1]) \end{cases}$$

(4p)

3. Let $T : X \to X$ be a mapping (not necessary linear) on a normed space X. Moreover assume that there are real constants C, α, where $\alpha > 1$, such that

$$\|T(x) - T(y)\| \le C\|x-y\|^\alpha, \text{ for all } x, y \in X.$$

Show that there exists a $z \in X$ such that $T(x) = z$ for all $x \in X$.

(4p)

4. State and prove Hilbert-Schmidt's theorem.

(5p)

5. Let A be a bounded operator on a Hilbert space H. Define the adjoint operator A^* (also prove that it exists) and show that A^* is a bounded operator on H with $\|A\| = \|A^*\|$.

(4p)

6. Let T be a self-adjoint operator on a Hilbert space H. Assume that T^n is compact for some integer $n \geq 2$. Prove that T is compact.

(4p)

Good Luck!!
PK