TMA 401/MAN 670 Functional Analysis 2002/2003
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First Assignment

hand in at latest on Friday April 11

1. Show that linear mappings preserve linear dependence. Do linear mappings preserve linear independence?

2. Define T as

$$Tf(x) = \int_0^x \sin(y) f(y)\, dy, \ x \in [0, 2\pi],$$

where $f \in \mathcal{C}([0, 2\pi])$. Calculate the operator norm $\|T\|$ where T is considered as a mapping $\mathcal{C}([0, 2\pi]) \to \mathcal{C}([0, 2\pi])$. Show that the integral equation

$$5f(x) + 2\cos x = Tf(x), \ x \in [0, 2\pi]$$

has a unique solution $f \in \mathcal{C}([0, 2\pi])$ and calculate the solution.

3. Show that all norm functions on finite-dimensional vector spaces are equivalent. Is this true for infinite-dimensional vector spaces?

4. Show that l^1 is a subspace in l^2. Show that this subspace is not a closed set in l^2, i.e. the closure of l^1 in l^2 is not equal to l^1.