CHALMERS | GÖTEBORGS UNIVERSITET
Matematiska Institutionen
Peter Kumlin

6th April 2004

TMA401 Functional Analysis
MAN670 Applied Functional Analysis
4th quarter 2003/2004

All document concerning the course can be found on the course home page:
http://www.math.chalmers.se/Math/Grundutb/CTH/tma401/

Home Assignment 1

Problem 1: Let Y be a finitedimensional subspace of a normed space X. Show that Y is closed.

Problem 2: Show that l^1 (as a vector space) is a subspace of l^2. Is this subspace closed in l^2 with the l^2-norm?

Problem 3: Let X be a normed space. Show that X is finitedimensional if and only if every closed and bounded set in X is compact.

Problem 4: Set $X = l^2$ with the $\| \; \|_{l^2}$-norm and define the mappings T_1, T_2 by

$$T_1(x_1, x_2, x_3, \ldots, x_n, \ldots) = (x_1, \frac{1}{2}x_2, \frac{1}{3}x_3, \ldots, \frac{1}{n}x_n, \ldots)$$

and

$$T_2(x_1, x_2, x_3, \ldots, x_n, \ldots) = (x_1, x_2^2, x_3^3, \ldots, x_n^n, \ldots)$$

for $(x_1, x_2, x_3, \ldots, x_n, \ldots) \in l^2$. Is T_1 a linear mapping? Is T_2 a linear mapping? Is T_1 continuous at any point in l^2? Is T_2 continuous at any point in l^2? Calculate

$$\sup\{\|T(x_1, x_2, x_3, \ldots, x_n, \ldots)\|_{l^2} : \|(x_1, x_2, x_3, \ldots, x_n, \ldots)\|_{l^2} \leq r\}$$

for all $r > 0$ for both T equal to T_1 and to T_2. Explain the difference.

Problem 5: Let X be a Banach space and let $T_n \in \mathcal{B}(X, X)$, $n = 1, 2, 3, \ldots$ Assume that $\lim_{n\to\infty} T_n x$ exists for every $x \in X$. Show that $T \in \mathcal{B}(X, X)$ where T is defined by

$$Tx = \lim_{n\to\infty} T_n x$$

for $x \in X$.

The solutions should be handed in at the latest on **Friday April 30**.