CHALMERS | GÖTEBORGS UNIVERSITET

Matematiska Institutionen
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17th April 2005

TMA401 Functional Analysis
MAN670 Applied Functional Analysis
4th quarter 2004/2005

All document concerning the course can be found on the course home page:
http://www.math.chalmers.se/Math/Grundutb/CTH/tma401/

Home Assignment 1

Problem 1: Let X be a vector space and let $\|x\|$ and $\|x\|_*$, $x \in X$, be two norms on X. Is $\max(\|x\|, \|x\|_*)$ a norm on X? Is $\min(\|x\|, \|x\|_*)$ a norm on X?

Problem 2: Consider the vector space l^1 and set $\|\mathbb{x}\|_* = 2|\Sigma_{n=1}^{\infty} x_n| + \Sigma_{n=2}^{\infty}(1 + \frac{1}{n})|x_n|$ for $\mathbb{x} = (x_1, x_2, \ldots, x_n, \ldots) \in l^1$. Show that $\|\mathbb{x}\|_*$ defines a norm on l^1 and that the vector space l^1 is a Banach space with this norm. Is this norm equivalent to the standard norm $\|\mathbb{x}\|_{l^1}$?

Problem 3: Let T be defined by $T(\mathbb{x}) = (x_2, x_3, \ldots, x_{n+1}, \ldots)$ for all $\mathbb{x} = (x_1, x_2, \ldots, x_n, \ldots) \in l^2$. Show that $T \in \mathcal{B}(l^2, l^2)$ and calculate $\|T\|$.

Problem 4: Let X be a Banach space and let T, S be two mappings from X into X (not necessarily linear). Assume that $TS = ST$ and that T has a unique fixed point. Show that S has a fixed point. What can be said if T has more than one fixed point?

Problem 5: Let F be a compact set in a normed space X and let $T : F \to F$ have the property

$$\|T(x) - T(y)\| < \|x - y\|, \quad \text{all } x \neq y \in F.$$

Show that T has a unique fixed point.

Problem 6: Let X be a Banach space and T a mapping on X satisfying

$$\|T(x) - T(y)\| \geq K\|x - y\| \;\text{ all } x, y \in X,$$

where $K > 1$. Assume that $T(X) = X$. Show that T has a unique fixed point.

The solutions should be handed in at the latest on **Friday April 29**.