Matematik, CTH & GU
Tentamen i Funktionalanalys TMA401/MAN670
Hjälpmedel: Inga (inte ens räknedosa).
Personuppgifter: Namn, personnummer, linje, antagningsår.
Inlämning ska ske i uppgifternas ordning; v.g. sidnumrera!
Teoriuppgifter: 4,5,6.
Telefon: Peter Kumlin 035-52077 eller 0739603800

Datum: 2004–05–29
Skrivtid: fm (5 timmar)

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1. Show that the following boundary value problem

$$\begin{cases} u''(x) - u(x) + \dfrac{1}{2}(1 + u(x^2)) = 0, & 0 \le x \le 1 \\ u(0) = u'(0) = 0, & u \in C^2([0,1]) \end{cases}$$

has a unique solution.

(4p)

2. Let $H = L^2([a,b])$, a, b finite, and

$$Tf(x) = \frac{1}{b-a}\int_a^b f(x)\,dx, \quad x \in [a,b].$$

Show that T is a bounded linear operator $H \to H$ and that T is a projection.

(4p)

3. Let $h \in C([0,1] \times [0,1])$ be a real-valued function such that

$$h(x,y) = h(y,x) > 0$$

for all $x, y \in [0,1]$. Set

$$Tf(x) = \int_0^1 h(x,y)f(y)\,dy, \quad x \in [0,1]$$

for $f \in L^2([0,1])$. Show that the bounded linear operator T on $L^2([0,1])$ has an eigenvalue $\lambda = \|T\|$ which is simple.

(4p)

4. State and prove the Orthogonal Projection theorem[1]. Also the "Closest Point Property" theorem should be proved.

(5p)

5. Define the notion of weak convergence on a Hilbert space and show that every weakly convergent sequence is bounded.

(4p)

6. Show that for every compact self-adjoint operator T on a Hilbert space there exists an eigenvalue λ of T with $|\lambda| = \|T\|$. Also show that there can be no eigenvalue μ of T with $|\mu| > \|T\|$.

(4p)

Good Luck!!
PK

[1] Often referred to as the Orthogonal Decomposition theorem.