Matematik, CTH & GU

Tentamen i Funktionalanalys TMA401/MAN670

Hjälpmedel: Inga (inte ens räknedosa).

Personuppgifter: Namn, personnummer, linje, antagningsår.

Inlämning ska ske i uppgifternas ordning; v.g. sidnumrera!

Teoriuppgifter: 4,5,6.

Telefon: Peter Kumlin 031-7723532, 0739-603800

Lärare besöker skrivsalen ca 9.30 och 11.30.

Datum: 2005–05–25

Skrivtid: fm (5 timmar)

Lokal: V

---

1. Show that the boundary value problem

$$\begin{cases} u''(x) + u(x) + \lambda \cos(1 + u(x)) = 0, & 0 \leq x \leq 1 \\ u(0) = u'(0) = 0, & u \in C^2([0,1]) \end{cases}$$

has a unique solution for $|\lambda| \leq \epsilon$, ϵ small. Give an upper bound on ϵ .

(4p)

2. Let $(e_n)_{n=1}^{\infty}$ be an ON-basis in a Hilbert space H and define the operator T by

$$T(\Sigma_{n=1}^{\infty} a_n e_n) = \Sigma_{n=2}^{\infty} \frac{1}{n} a_n e_{n-1}.$$

Show that T is compact and find T^*. Find[1] $\sigma_p(T)$ and $\sigma_p(T^*)$.

(4p)

3. Let $f_1, f_2, \ldots, f_n \in \mathcal{B}(H, \mathbb{C})$ be linearly independent where H is a Hilbert space. Show that there exist $x_1, x_2, \ldots, x_n \in H$ such that

$$f_i(x_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

for all $i = 1, 2, \ldots, n$.

(4p)

[1] $\sigma_p(A) = \{\lambda : \lambda \text{ eigenvalue to } A\}$.

4. State and prove the Orthogonal Projection theorem[2]. Also the "Closest Point Property" theorem should be proved.

(5p)

5. Let $T \in \mathcal{B}(X, X)$ where X is a Banach space and $\|T\| < 1$. Show that $I + T$ is an invertible operator, i.e. $(I + T)^{-1} \in \mathcal{B}(X, X)$.

(4p)

6. Let $T : H \to H$ be a linear mapping in a Hilbert space H. Assume that $Tx_n \rightharpoonup Tx$ for every $x_n \to x$. Show that $T \in \mathcal{B}(H, H)$.

(4p)

Good Luck!!
PK

[2]Often referred to as the Orthogonal Decomposition theorem.