CHALMERS | GÖTEBORGS UNIVERSITET

Matematiska Institutionen
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TMA401 Functional Analysis
MAN670 Applied Functional Analysis
4th quarter 2006/2007

All document concerning the course can be found on the course home page:
http://www.math.chalmers.se/Math/Grundutb/CTH/tma401/

Home Assignment 2

Problem 1: Let $(x_n)_{n=1}^{\infty}$ be an ON-basis in a Hilbert space H and let $(y_n)_{n=1}^{\infty}$ be another ON-sequence such that

$$\Sigma_{n=1}^{\infty} \|x_n - y_n\|^2 < 1.$$

Show that $(y_n)_{n=1}^{\infty}$ is an ON-basis.

Problem 2: Let $x_n = (0, 0, \ldots, 0, 1, 2, 0, \ldots)$ where the numbers 1 and 2 appear in the positions n and $n+1$ and let $y_n = (1, 1, \ldots, 1, 0, 0, \ldots)$ with the number 1 in the first n positions. Consider these as vectors in l^2. Prove that for all $n = 1, 2, \ldots$

$$y_n \notin \overline{\text{Span}\{x_1, x_2, \ldots\}}.$$

Problem 3: Prove the existence and uniqueness of a solution to the following boundary value problem:

$$\begin{cases} 4u''(x) = |x + u(x)|, & 0 \le x \le 1 \\ u(0) - 2u(1) = u'(0) - 2u'(1) = 0, & u \in C^2([0,1]). \end{cases}$$

Problem 4: Set $H = L^2([0,1])$. Let T be given by

$$Tf(x) = \int_{1-x}^{1} f(t)\, dt.$$

Show that

1. T is a bounded linear operator on H and
2. calculate the kernel $k(x,t)$ for T and show that T is self-adjoint.
3. Moreover calculate the kernel $k_2(x,t)$ for T^2 and
4. find[1] all eigenvalues and eigenfunctions for T.
5. Finally calculate $\|T\|$.

The solutions should be handed in at the latest on **Friday May 25**.

[1]Hint: Let f be an eigenfunction for T and calculate $(T^2 f)''$. Show that f is a solution to the equation $\lambda^2 f'' + f = 0$.