TMA401/MMA400 Functional Analysis 2007/2008
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Homework assignment 2

Deadline 2007-10-05

Problem 1: Construct an inner product space $(E, \langle \cdot, \cdot \rangle)$ and a closed subspace $F \subset E$, $F \neq E$ such that $F^{\perp} = \{0\}$.

Problem 2: Define an inner product on $\mathbb{C}^4$ by

$$\langle (z_0, z_1, z_2, z_3), (v_0, v_1, v_2, v_3) \rangle = \int_0^1 f(t)\overline{g(t)}\, dt$$

where $f(t) = z_0 + z_1 t + z_2 t^2 + z_3 t^3$ and $g(t) = v_0 + v_1 t + v_2 t^2 + v_3 t^3$. Show that $\langle \cdot, \cdot \rangle$ is indeed an inner product on $\mathbb{C}^4$ and calculate a formula for the induced norm.

Problem 3: Let $x_1, x_2, \ldots, x_N$ be linearly independent vectors in an inner product space with $N = \begin{pmatrix} n+1 \\ 2 \end{pmatrix}$. Show that there are orthonormal vectors $y_1, y_2, \ldots, y_n$ such that

$$y_k = \Sigma_{i \in A_k} \alpha_i x_i, \quad k = 1, 2, \ldots, n$$

where $A_1, A_2, \ldots, A_n$ form a partition of the set $\{1, 2, \ldots, N\}$.

Problem 4: Prove or give a counter example to the statement: $x_n \rightharpoonup x$ in E and $\alpha_n \to \alpha$ in $\mathbb{C}$ implies that $\alpha_n x_n \rightharpoonup \alpha x$ in E. Here E is an inner product space.