TMA401/MMA400 Functional Analysis 2007/2008

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Homework assignment 3

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Problem 1: Consider the ODE

$$u''(x) - u(x) + \frac{1}{2}(1 + u(x^2)) = 0,\ x \in [0,1],$$

with the boundary conditions $u(0) = u'(0) = 0$. Calculate the Green's function to this BVP and show that the problem has a unique C^2-solution $u(x)$.

Problem 2: Let $k \in C([0,1] \times [0,1])$ and set

$$A(f)(x) = \int_0^1 k(x,t) f(t)\, dt,\ x \in [0,1],$$

for $f \in C([0,1])$. Show that $A : C([0,1]) \to C([0,1])$ is a compact operator.

Problem 3: Let H be a complex Hilbert space and let $A : H \to H$ be a bounded linear operator satisfying

$$\langle A(x), x \rangle \in \mathbb{R} \text{ for all } x \in H.$$

Show that A is self-adjoint.

Problem 4: Let $(x_n)_{n=1}^{\infty}$ be a bounded sequence in a Hilbert space H. Moreover let $A : H \to H$ be a bounded linear operator on H. Show that if $(A^*A(x_n))_{n=1}^{\infty}$ converges in H then also $(A(x_n))_{n=1}^{\infty}$ converges in H