TMA401/MMA400 Functional Analysis 2012/2013

Peter Kumlin
Mathematics
Chalmers & GU

Homework assignment 3

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Problem 1 : Set

$$A(f)(x) = \int_0^\pi \sin(x-y)f(y)\,dy, \quad 0 \le x \le \pi.$$

Find the norm of A regarded as an operator on

1. the Banach space $C([0,\pi])$,
2. the Hilbert space $L^2([0,\pi])$.

Problem 2 : Let E be a Hilbert space with an ON-basis $(e_n)_{n=1}^\infty$. Set

$$A(x) = \Sigma_{n=1}^\infty \langle x, e_n\rangle e_{n+1},\ x \in E.$$

What are the eigenvalues of A? Is A compact? Is A self-adjoint? What is the norm of A?

Problem 3 : Let E be a Hilbert space with an ON-basis $(e_n)_{n=1}^\infty$ and let A be a bounded linear mapping from E into E such that

$$\Sigma_{n=1}^\infty \|A(e_n)\|^2 < \infty.$$

Show that if $(f_n)_{n=1}^\infty$ is an other ON-basis for E then

$$\Sigma_{n=1}^\infty \|A(e_n)\|^2 = \Sigma_{n=1}^\infty \|A(f_n)\|^2.$$

Moreover show that

$$\|A\| \le (\Sigma_{n=1}^\infty \|A(e_n)\|^2)^{\frac{1}{2}}.$$

Problem 4 : Analyse existence and uniqueness for solutions to the following BVP:

$$\begin{cases} u''(x) - u(x) + \lambda \arctan(u(x^2)) = 0, & 0 \le x \le 1 \\ u(0) = 0, u(1) = 1, & u \in C^2([0,1]) \end{cases}$$

Here λ is a real number and all functions are real-valued. What can be said for different values of λ?