TMA401/MMA400 Functional Analysis 2018/2019

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Homework assignment 1

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Problem 1 : Let Y be a finite-dimensional subspace in an infinite-dimensional normed space $(E, \|\cdot\|)$. Show that Y is closed.

Problem 2 : Consider the normed space $C([0,1])$ with norm $\|f\| = \max_{x\in[0,1]} |f(x)|$. Set $M = \{f \in C([0,1]) : f \text{ is an increasing function}\}$. Show that

1. M is not an open set,
2. M is a closed set.

Problem 3 : Let $C^1([0,1])$ be the vector space of all continuously differentiable functions $f : [0,1] \to \mathbb{R}$. Show that

1. $C^1([0,1])$ with the norm $\|f\| + \|f'\|$ is a Banach space,
2. $C^1([0,1])$ with the norm $\|f\|$ is not a Banach space.

Here $\|f\|$ denotes $\max_{x\in[0,1]} |f(x)|$.

Problem 4 : Consider the normed space $C([0,1])$ with norm $\|f\| = \max_{x\in[0,1]} |f(x)|$. Assume that $T : C([0,1]) \to C([0,1])$ is a linear mapping with the property that $Tf(x) \geq 0$ for all $x \in [0,1]$ provided $f(x) \geq 0$ for all $x \in [0,1]$. Show that

1. T is continuous,
2. $\|T\| = \max_{x\in[0,1]} T\mathbf{1}(x)$ where $\mathbf{1}$ denotes the constant function taking the value 1.