

*Hjälpmedel:* Kurslitteratur, anteckningar och valfri räknare.

*Telefon:* Milena Anguelova tel.

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1. Express  $D(g\delta)(\varphi)$  in values of  $g$  and  $\varphi$  only. Here  $g$  is an admissible function and  $\varphi \in \mathcal{S}$ . (6 p)

2. Determine the Hilbert transform of the function  $f(x) = \sin x + \cos x$ , and the analytic function of which  $f$  is the real part. (6 p)

3. Show that, for the low pass filter function  $H$  with real coefficients in an orthogonal MRA,  $|H(\pi/2)| = 1/\sqrt{2}$ . (6 p)

4. Let  $\psi$  be the wavelet in an orthogonal MRA. Prove that, for  $\hat{f} = 1_{(a,b)}$  &  $(a, b) \subset (0, 2\pi)$ ,

$$(2\pi)^{-1} \int |\hat{f}(\omega)|^2 |\hat{\psi}(\omega)|^2 d\omega = \sum_k |\langle f, \psi_{0k} \rangle|^2$$

holds. (A consequence of this result is that the equality holds for all  $f \in L^2$ , and for any scale  $j$ .) (7 p)