

Proposed solutions FWA August '07

1. Factorization of the product filter gives H , which then gives G .
2. In the definition, write $e^{-2\pi i kn/N} = e^{-2\pi i k(n-l)/N} e^{-2\pi i kl/N}$.
3. The transform may be performed independently in each variable, and multiplication by the variable corresponds to differentiation.
4. For $s > 0$, $\int_0^\infty x^{-1/2} e^{-2\pi i s x} dx = s^{-1/2} \int_0^\infty x^{-1/2} e^{-2\pi i x} dx = s^{-1/2} 2 \int_0^\infty e^{-2\pi i x^2} dx = s^{-1/2} (1 - i)/2$. Thus, for $s \in \mathbb{R}$, the transform is $|s|^{-1/2} (1 - i \operatorname{sign} s)/2$.