MATEMATISKA VETENSKAPER, Chalmers & Göteborgs universitet
Hjälpmedel: Kurslitteratur, anteckningar, valfri räknare
Telefon: Magnus Goffeng, tel. 0762-721860

MMA410, TMA 462 Fourier and Wavelet Analysis

2007–12–19 kl. 8.30–13.30

---

1. Compute the Fourier transform of

$$f(x) = \begin{cases} 2x & 0 < x < a \\ 1-x & a < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   for $0 < a < 1$. (5p)

2. Prove that $|f * g| \leq \int_{-\infty}^{\infty} |FG| d\xi$, if F and G are the Fourier transforms of F and G respectively. How does this relation change depending on the definition of the Fourier transform that is used? (5p)

3. Write down a reasonable definition of an *even* tempered distribution, and prove that δ'' is even. (5p)

4. Let $\{\psi_{j,k}\}$ be an orthonormal basis in $L^2(\mathbb{R})$. Put $\Psi(t) = 2^{1/2}\psi(2t)$, and define $\Psi_{j,k}$ by rescaling and translation in the same way as the $\psi_{j,k}$ are obtained from the wavelet ϕ. Prove that $\{\Psi_{j,k}\}$ is an orthonormal system that is not a Riesz basis. (5p)

5. Let $f(x, y) = \text{sinc}(x)^2\text{sinc}(2y)^2$, and let $R_\theta f(r)$ be the Radon transform of f. Show that for a given angle θ, it is enough to sample $R_\theta f(r)$ at discrete points $\{r_j\}$. How densely must it be sampled? (5p)

Good luck!
Bernt W.