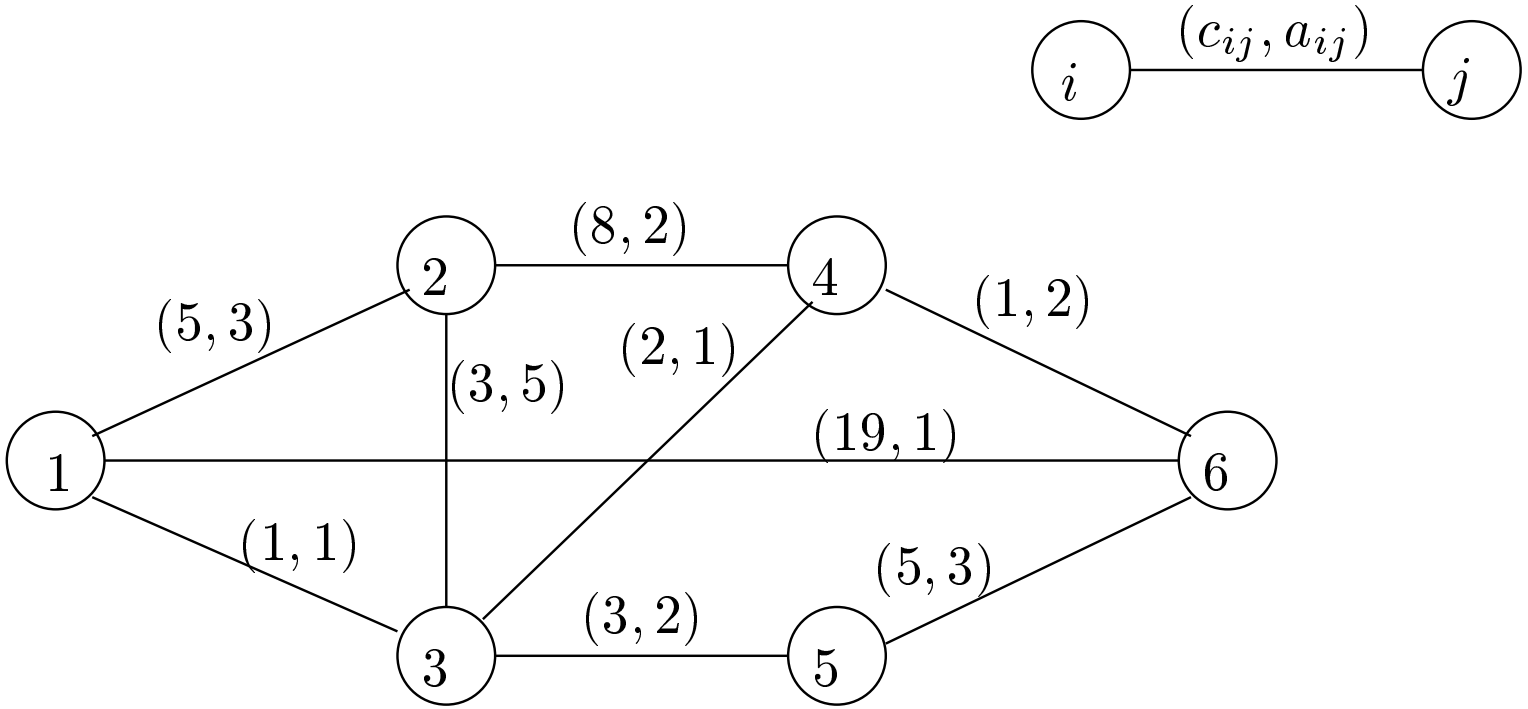
Questions on the network design problem

1. Formulate the minimum spanning tree problem (MST) as a network flow problem. [*Hint*: consider node 1 as a sink and all other nodes as sources with strength 1.]
2. Consider the graph below.



(a) Provide *all* the spanning trees of this graph explicitly. Calculate the sum of c_{ij} and a_{ij} for each tree. Which ones are feasible with respect to the *budget constraint*

$$\sum_{(i,j)\in\mathcal{T}} a_{ij} \leq 10$$

(where $\mathcal{T}$ denotes a collection of links forming a spanning tree)? Which ones are optimal (minimal) with respect to the link costs c_{ij}?

(b) Utilize the solution in (a) to formulate this problem for a general graph.

(c) Formulate the MST problem as a binary, integer programming problem.

(d) Is there a polynomial algorithm for the problem in (b)? [*Hint*: utilize that the binary knapsack problem is hard.]

3. Provide a polynomial *heuristic* for the problem which gives a feasible solution.
4. Provide a *local search* heuristic which improves a feasible solution.
5. Provide a *Lagrangian relaxation* algorithm.
   (a) Suggest a suitable relaxation.
   (b) How are the subproblems solved?
   (c) Suggest a primal feasibility heuristic.
   (d) Provide a complete Lagrangian relaxation scheme.
6. Suggest a *Branch & Bound* algorithm.
   (a) Suggest a suitable Lagrangian relaxation.
   (b) Suggest a proper branching rule.
   (c) Provide a complete B & B algorithm.
7. Apply some of these algorithms on the above example.