Opportunistic maintenance optimization of multicomponent systems with deterministic and stochastic lives

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Introduction

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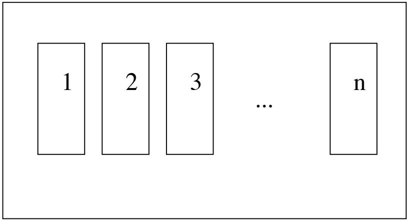
- There is a large potential for improvement!

We consider a systems with n components



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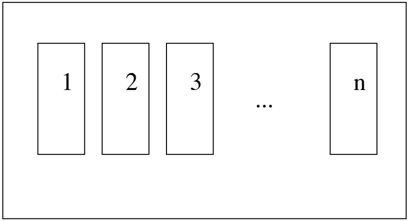

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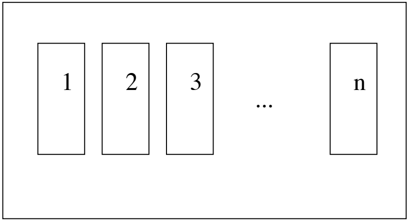

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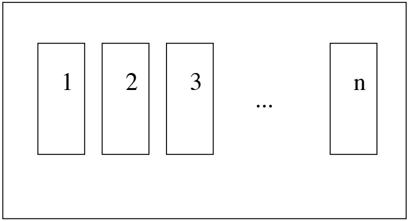

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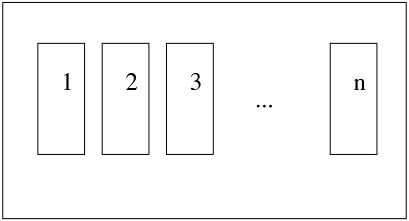

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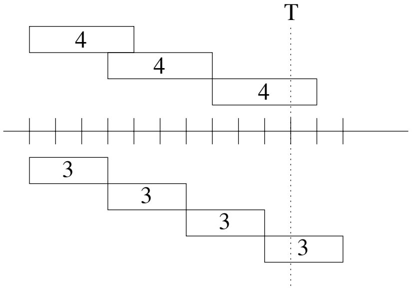
- A maintenance occasion costs d.
- Replacing a component i costs c_i.
- We want to find the minimal cost replacement schedule over a finite time horizon T.

Applications

- Energy industry (wind power, nuclear power, ...)
- Aircraft industry (Volvo Aero)
- Pulp production
- ...

The deterministic replacement problem



$$\text{cost} = 2c_1 + 3c_2 + 4d$$

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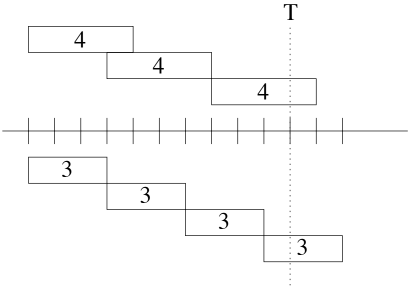

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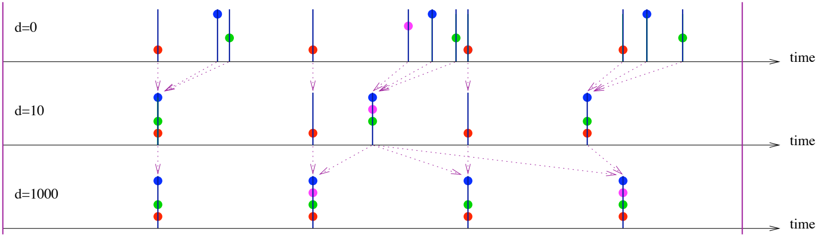
Definition

Given lives T_i for every component i,
costs c_{it}, d and timehorizon T,
minimize the maintenance cost.

Small example



The variables

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The deterministic model

the objective function

$$\text{minimize} \sum_t \left(\sum_i c_{it} x_{it} + d z_t \right)$$

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$$\begin{aligned} x_{it} &\leq z_t \\ \sum_{t=l+1}^{l+T_i} x_{it} &\geq 1, \quad l = 0, \ldots, T - T_i \end{aligned}$$

NP, P and NPC.

- NP: decision problems verifiable in polynomial time.

Example (set covering decision problem)

Given: $A = \{1, \ldots, k\}$, $S_1, \ldots, S_l \subset A$.
Question: Is there cover of cardinality $<= N$?

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- NPC: If all problems in NP are polynomially reducible to problem A, A is in NPC.

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NP-hard

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$\iff$ B is NP-hard

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- Given: $A = \{1, \ldots, k\}$, $S_1, \ldots, S_l \subset A$.
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Example (set covering optimization problem)

- Given: $A = \{1, \ldots, k\}$, $S_1, \ldots, S_l \subset A$.
  Question: Which is the cover of smallest cardinality?
- IP formulation: $a_{ij} = 1$ if $j \in S_i$ $a_{ij} = 0$ otherwise

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{l} y_i \\ \text{subject to} \quad & \sum_{i=1}^{l} a_{ij} y_i \geq 1 \quad j \in \{1, \ldots, k\} \\ & y_i \in \{0, 1\} \end{aligned}$$

Property I: the replacement problem is NP-hard.

Theorem

Set covering is polynomially reducible to the replacement problem.

Proof.

- Consider the replacement problem with $n = k$, $T = l, T_i = l$, $d = 1$, $c_{it} = 0$ if $i \in S_t$ and $c_{it} = 2$ otherwise.

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If RP in P then P=NP.

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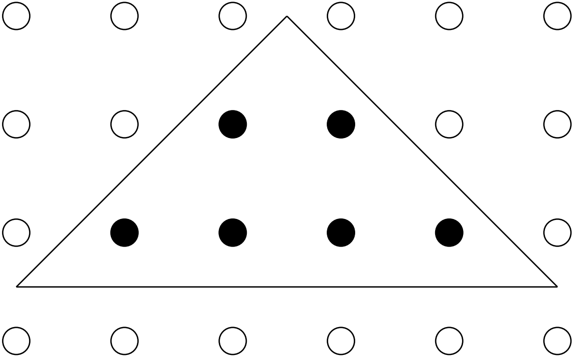
- Constraint matrix TU + integer r.h.s. $\Rightarrow$ integer polyhedron.
- Consecutive ones + unit matrix $\Rightarrow$ TU.

Property III: all inequalities are facet defining.

No inequalities are facett defining



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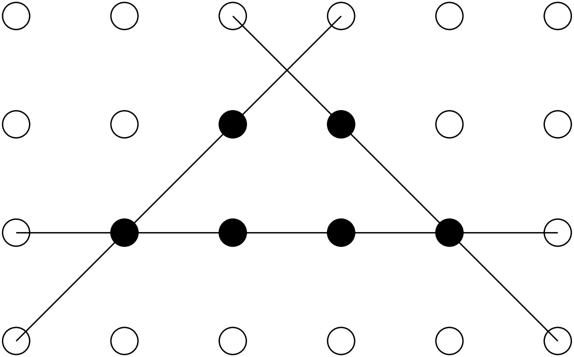

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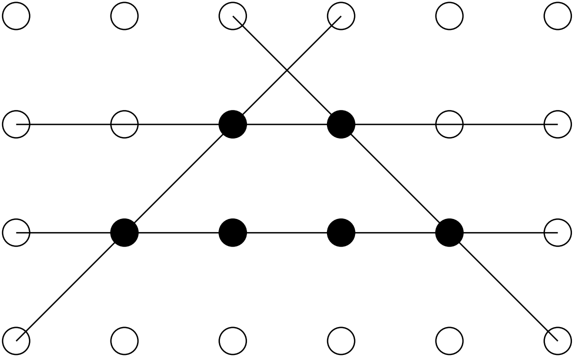
Integral polyhederon



The stochastic problem

- Only distributions of lives are known
- Can not creat an optimal schedule
- Can only minimze the expected cost

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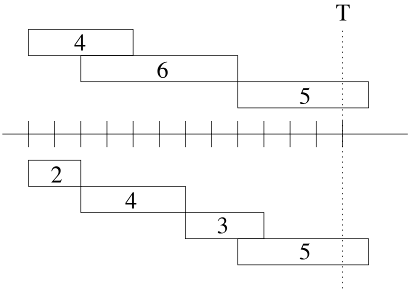
Definition

After the failure of some component, decide which additional components to replace in order to minimize the expected maintenance cost for the remaining planning horizon.

The one scenario problem



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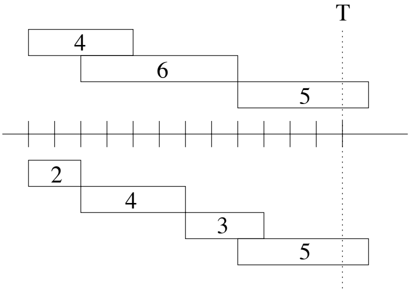

Definition

Given lives T_{ir} for every individual r of every component i, costs c_i, d and timehorizon T, minimize the maintenance cost.

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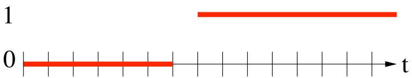

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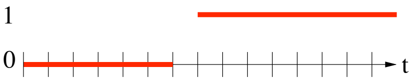

$$z_t = \begin{cases} 1 & \text{maintenance performed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

The one scenario model

the objective function

$$\text{minimize} \sum_i \left(c_i x_{i0}^1 + \sum_{t,r} c_i (x_{it}^r - x_{it-1}^r) + \sum_t dz_t \right)$$

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$$x_{it}^r \quad \leq \quad x_{it+1}^r$$

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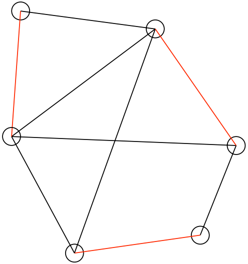
\sum_r x_{it}^r - x_{it-1}^r &\leq z_t \\
x_{it}^r &\leq x_{it+T_{ir+1}}^{r+1}
\end{aligned}$$

Properties

- The problem is NP-hard (reduction from vertex cover).



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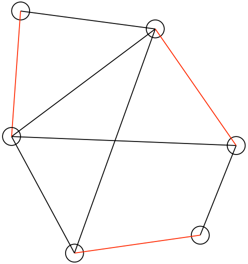

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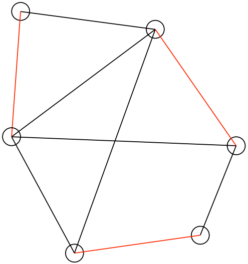

- We can relax the integrality on x_{it}^r.
- All inequalities are facet defining.

Two stage model

$$
\begin{aligned}
\text{minimize} \quad & \sum_{\omega} p(\omega)\left(\sum_{i}\left(c_i x_{i0}^{1\omega} + \sum_{t,r} c_i(x_{it}^{r\omega} - x_{it-1}^{r\omega}) + \sum_{t} d z_t^{\omega}\right)\right) \\
\text{subject to} \quad & x_{it}^{r\omega} \leq x_{it+1}^{r\omega} \\
& x_{it+1}^{r+1\omega} \leq x_{it}^{r\omega} \\
& x_{i0}^{r\omega} = 0, \quad r = 2, \ldots \\
& \sum_{r} x_{it}^{r\omega} - x_{it-1}^{r\omega} \leq z_t^{\omega} \\
& x_{it}^{r\omega} \leq x_{it+T_{ir+1}^{\omega}}^{r+1\omega} \\
& x_{i0}^{r\omega_1} = x_{i0}^{r\omega_2}
\end{aligned}
$$

Test case

- Wind power turbine with 5 major components

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- Simulation on 100 scenarios

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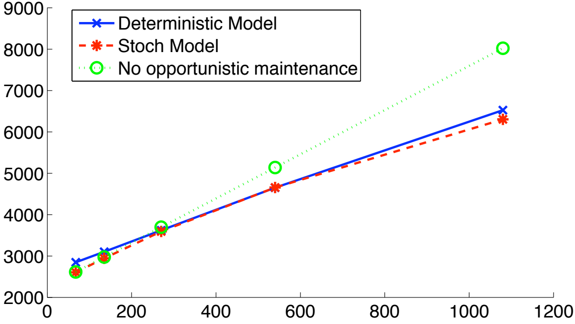
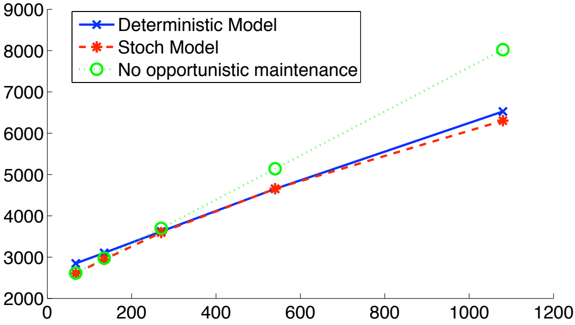
- Simulation on 100 scenarios
- $T = 25$ years, $d = 270$

component	median life	β	c
blades	200	1	270
pitch bearing	10	3.5	300
main bearing	14	3.5	480
gearbox	17	3.5	640
generator	14	3.5	190

Results


Deterministic Model
Stoch Model
No opportunistic maintenance
9000
8000
7000
6000
5000
4000
3000
2000
0
200
400
600
800
1000
1200

Results



- \$102 500 or 2.7 % improvement (stochastiv vs non-opportunistic)
- \$26 500 or 0.7 % improvement (stochastiv vs deterministic)

Conclusions

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Future research

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Future research

- We need an efficient method to solve the stochastic problem (Two stage and multi stage)
- Theoretical work:
  - New facets
  - Complexity of deterministic problem with time indepedent costs