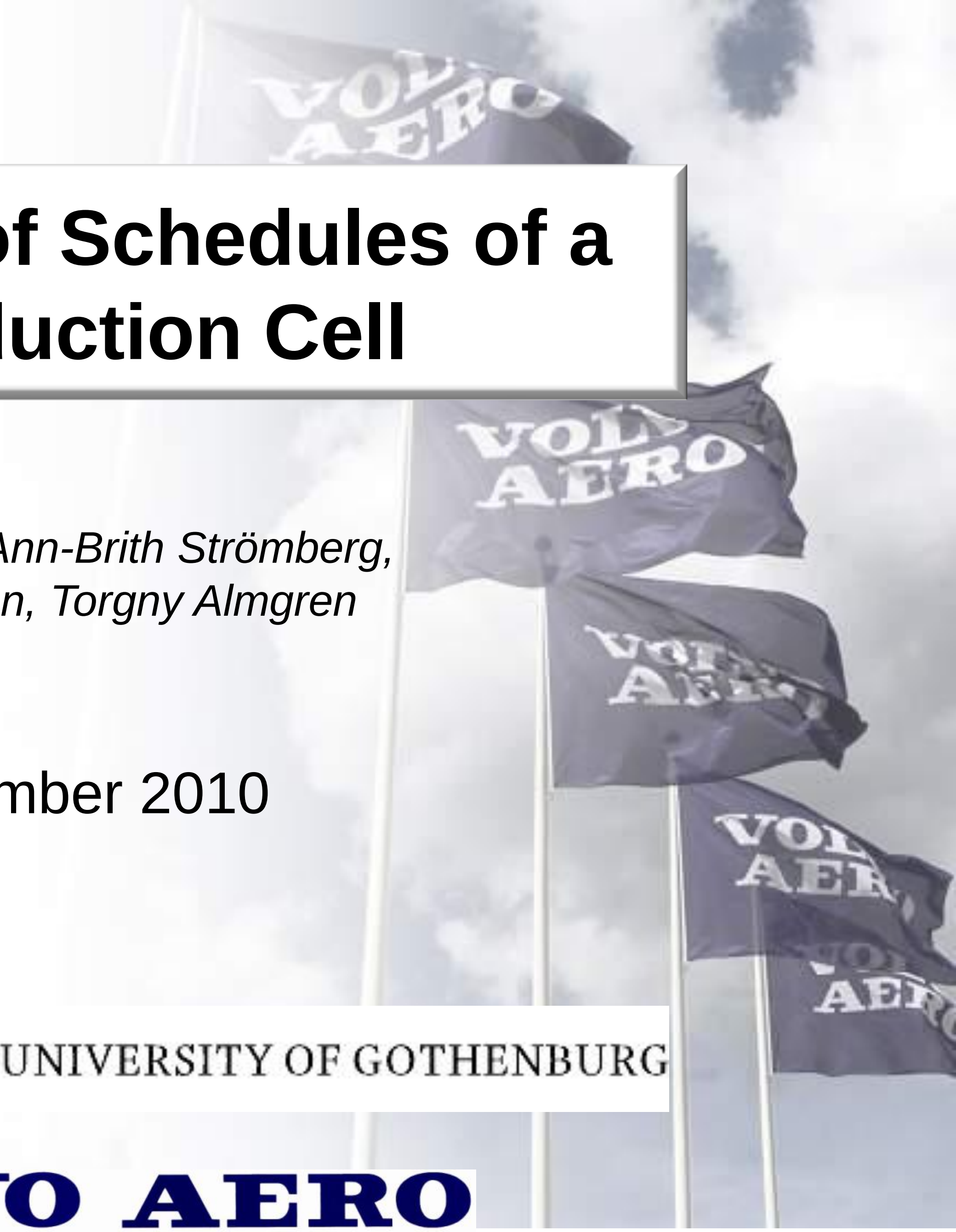

Optimization of Schedules of a Multitask Production Cell

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September 2010

CHALMERS

UNIVERSITY OF GOTHENBURG

VOLVO AERO

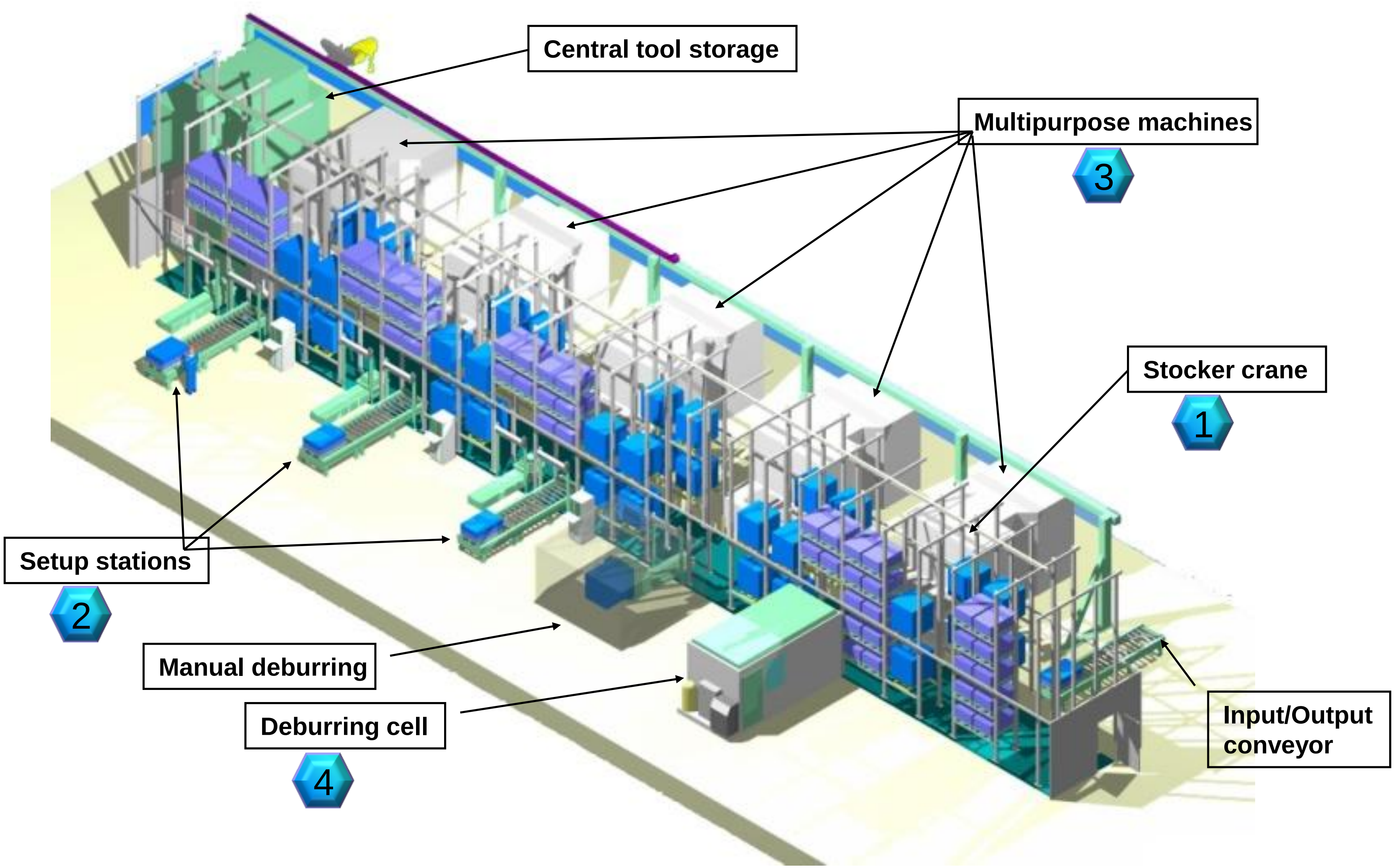
VOLVO AERO

- Part of Volvo Group
- Develops and produces aircraft and rocket engine components
- About 3000 employees

Parts processed in the Multi Task Cell

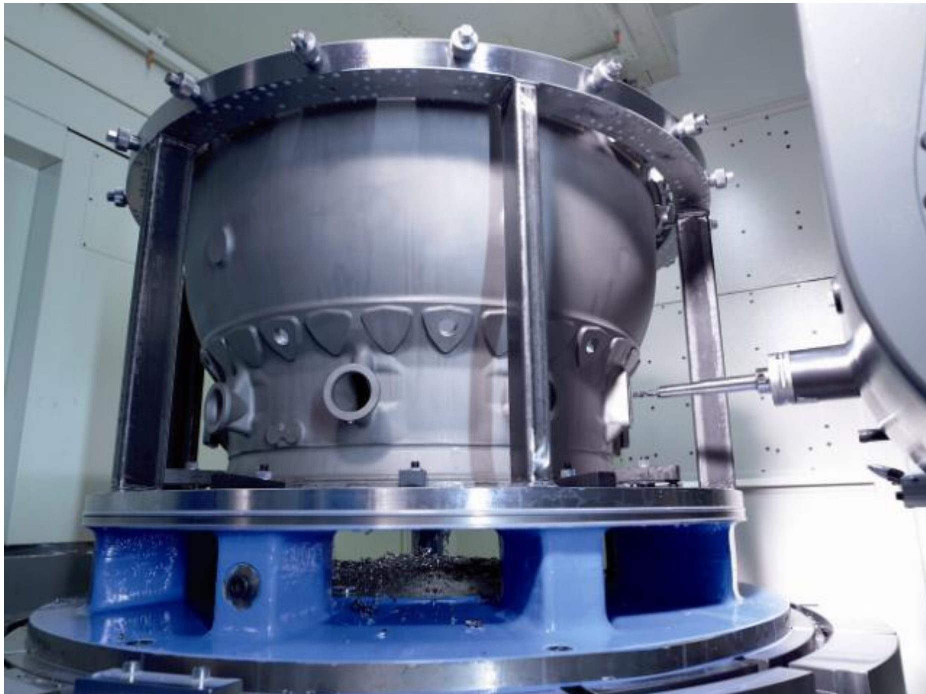
~ 8 compressor rear frames for different aero engines and gas turbines.
About 30 different jobs are processed in the Multi Task Cell.

The Multitask Production Cell

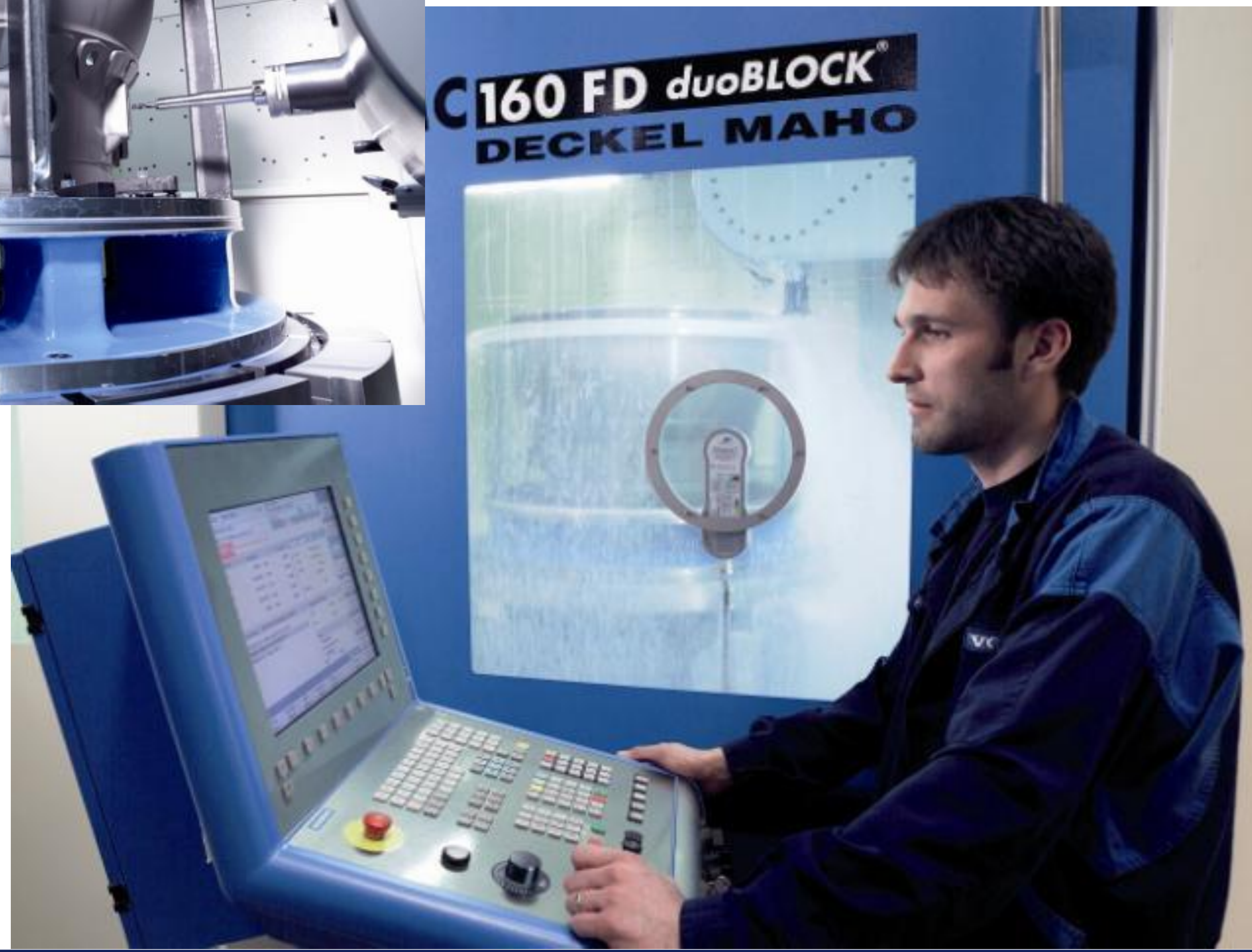


Stocker crane
Transports between storage and route operations

3 setup stations
Mount/demount in and out of fixtures



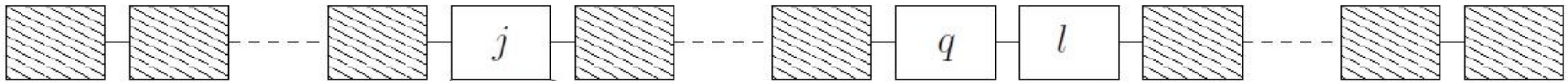
5 multitask machines
Drilling, milling and turning

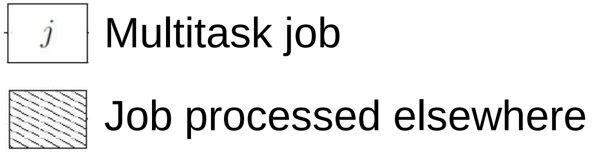


Automatic deburring cell
Robot deburring

The routing of a part

Every production order follows a routing in the planning system





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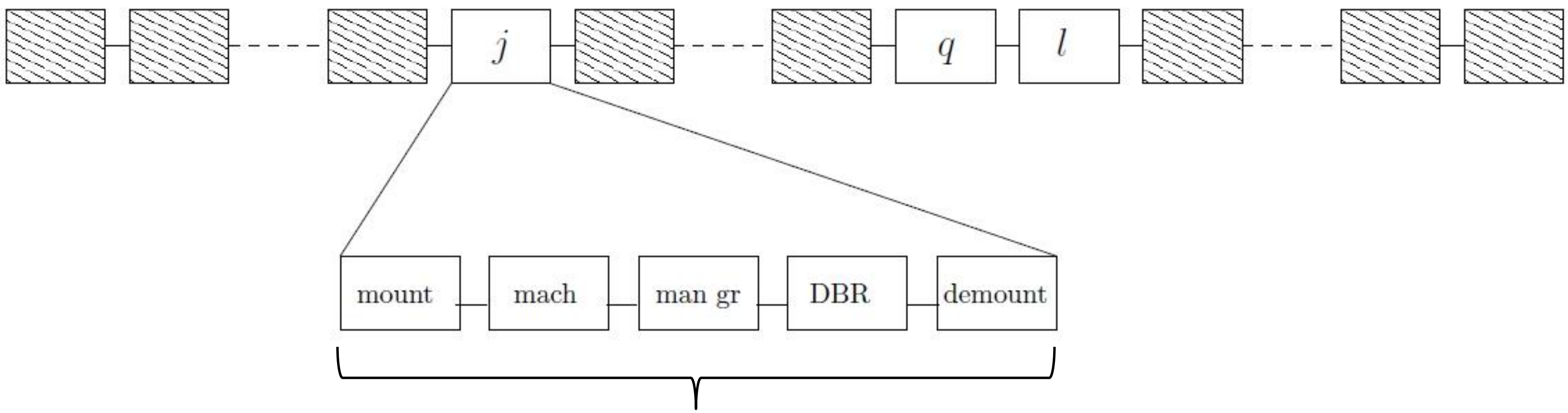
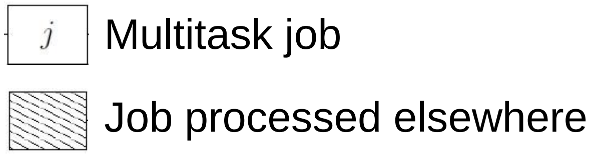
The routing of a part

Every production order follows a routing in the planning system



One **job** in the multitask cell ↔ 3-5 route operations



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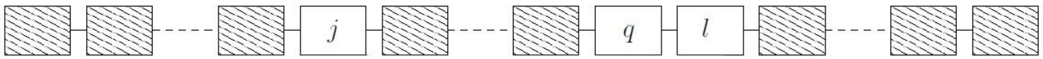
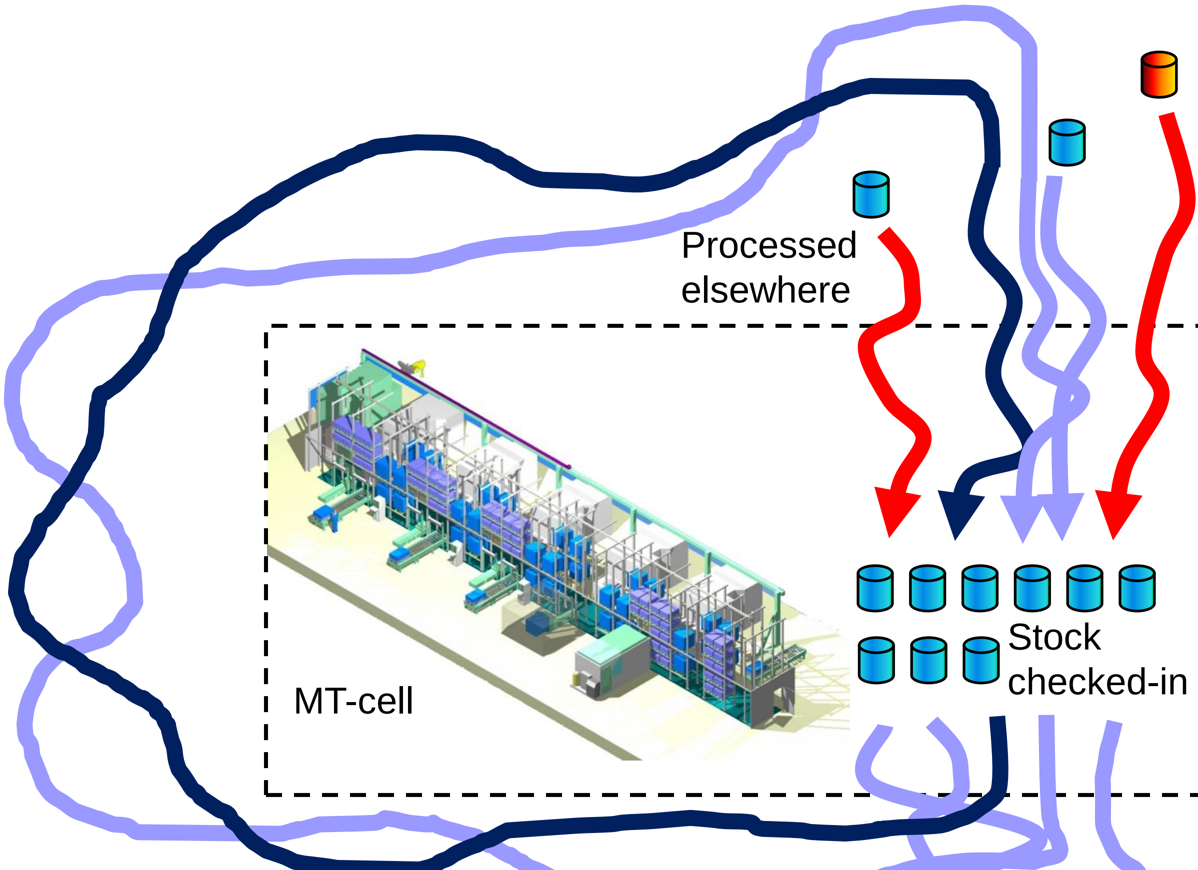
The queue of parts





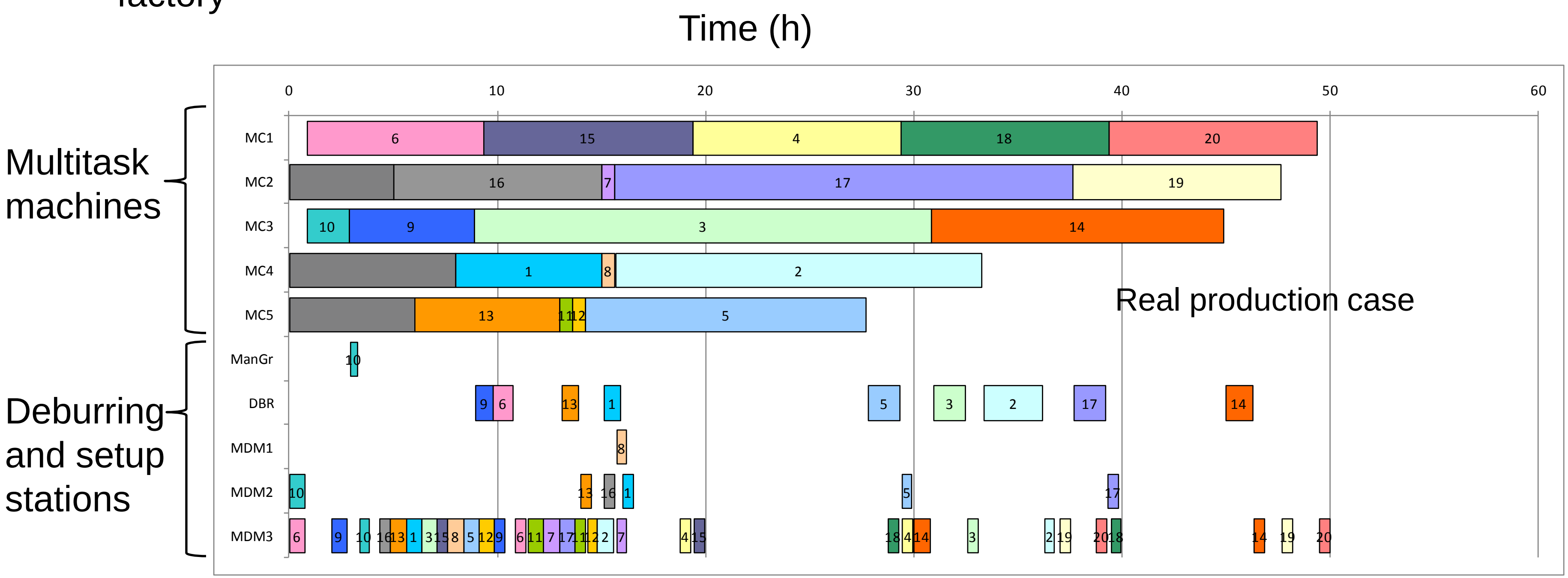
v_{0j}, planned lead time from current position to arrival at MT-cell

v_{jq} , planned lead time from completion of job j to arrival at MT-cell for job q

Current detail planning of the multitask cell

Manual planning based on

- Earliest Due Date priority list
- Other priorities based on the current logistical situation
- The FIFO priority rule (First In First Out) is used in other parts of the factory



The route operations of the remaining resources are set in a feasible schedule.

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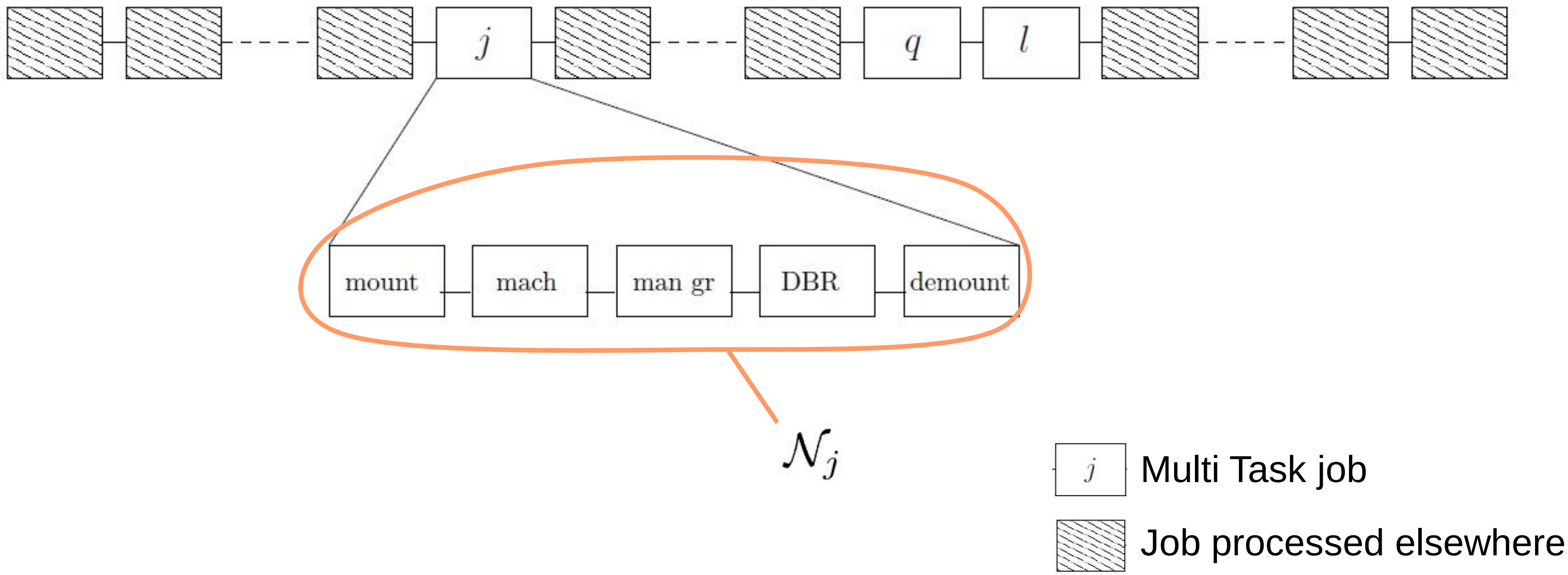
Sets and indices

$$i \in \mathcal{N}_j = \{1, ..., n_j\}, \text{ set of route operations}$$
$$j \in \mathcal{J}, \text{ set of jobs}$$
$$(j, q) \in \mathcal{Q}, \text{ set of pairs of subsequent jobs}$$
$$k \in \mathcal{K}, \text{ set of resources}$$



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Sets and indices

$$i \in \mathcal{N}_j = \{1, ..., n_j\}, \text{ set of route operations}$$
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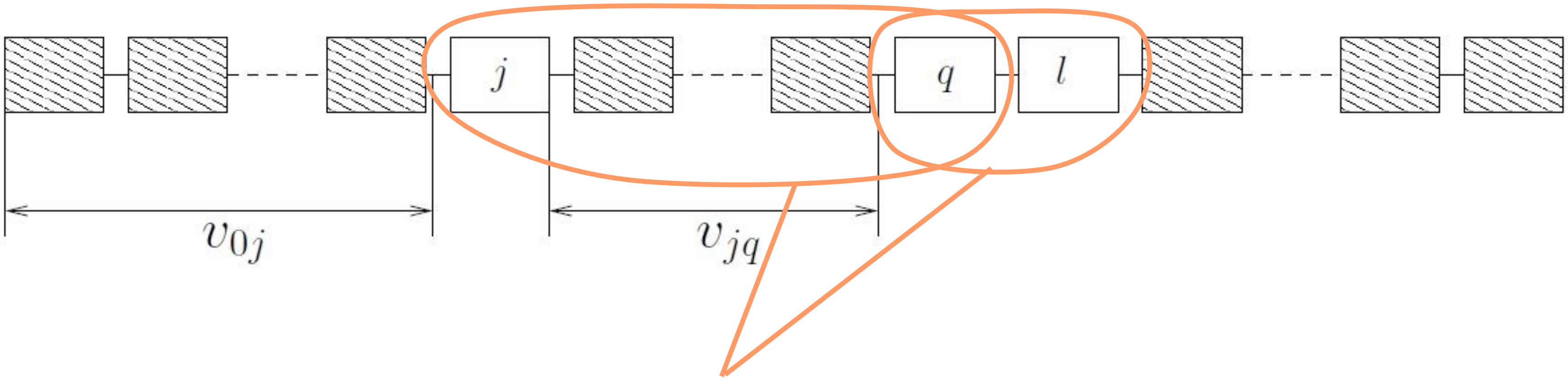
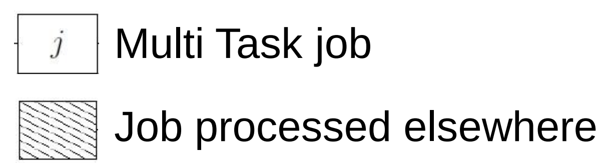


$$(j, q) \text{ and } (q, l) \in Q$$

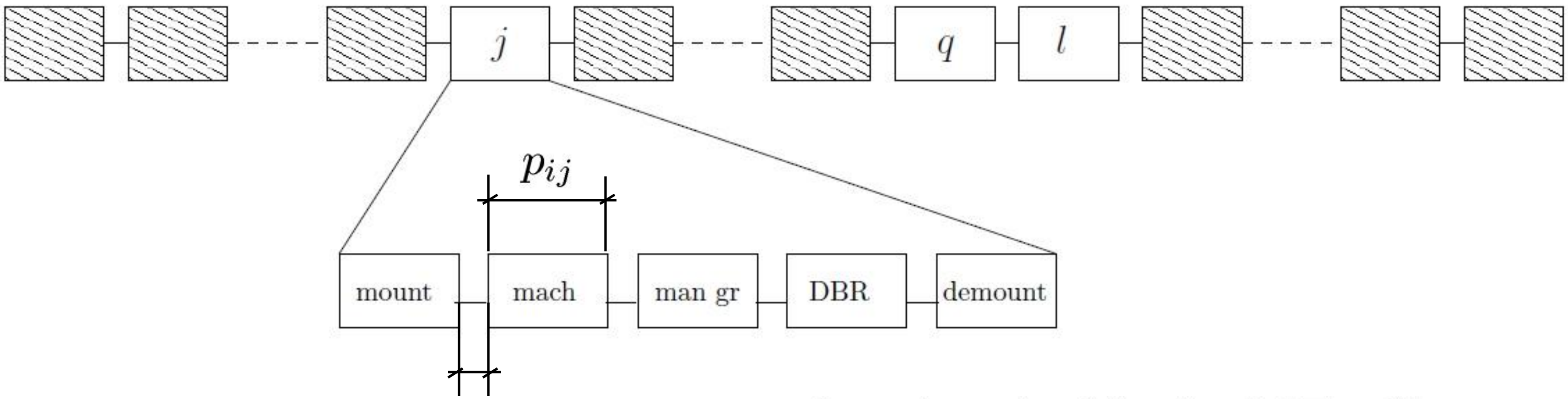


Parameters

$$\lambda_{ijk} = \begin{cases} 1, & \text{if op } (i,j) \text{ can be processed on } k \\ 0, & \text{otherwise} \end{cases}$$

a_k : k available the first time

p_{ij} : processing time for op (i,j)



w : transportation time inside the MT-cell

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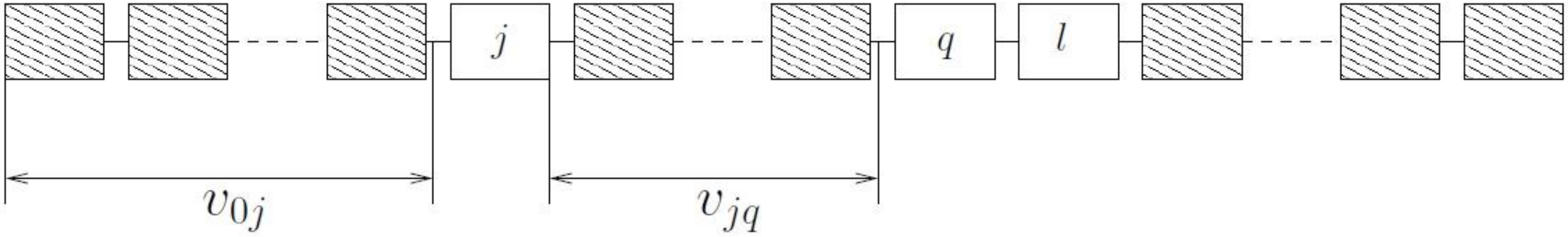
Parameters cont'd

r_j : release date for job j
d_j : due date of job j
v_{jq} : planned lead time between job j and job q, $(j,q) \in \mathcal{Q}$



If order checked-in: $r_j = r_q = r_l = 0$
Else: $r_j = r_q = r_l = max$ (date available (v_{0j}); planned release date)

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Variables

- Binary variables

$$z_{ijk} = \begin{cases} 1, & \text{if op } (i,j) \text{ scheduled on } k, \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ijpqk} = \begin{cases} 1, & \text{if op } (i,j) \text{ processed before op } (p,q) \text{ on } k, \\ 0, & \text{otherwise.} \end{cases}$$

- Time variables

$$t_{ij} = \text{starting time}$$

$$s_j = t_{n_j,j} + p_{n_j,j}, \text{ completion time of job } j.$$

$$h_j = \begin{cases} s_j - d_j, \text{ if positive, i.e. the tardiness of job } j, \\ 0, \text{ otherwise.} \end{cases}$$

The optimization model of the MT-cell

$$\text{Minimize} \quad \sum_{j \in J} (s_j + Ah_j)$$

$$\text{subject to} \quad \sum_{k \in \mathcal{K}} z_{ijk} = 1,$$

$$z_{ijk} \leq \lambda_{ijk},$$

$$y_{ijpqk} + y_{pqijk} \leq z_{ijk}, \qquad (i,j) \neq (p,q),$$

$$y_{ijpqk} + y_{pqijk} + 1 \geq z_{ijk} + z_{pqk}, \qquad (i,j) \neq (p,q),$$

$$t_{ij} + p_{ij} - M(1 - y_{ijpqk}) \leq t_{pq}, \qquad (i,j) \neq (p,q),$$

To be cont'd…

The optimization model of the MT-cell

weight (A=1)

$$\text{Minimize} \quad \sum_{j \in J} (s_j + A h_j)$$

The sum of completion times and tardiness, i.e. every job is done as early as possible and tardiness is punished.

$$\text{subject to} \quad \sum_{k \in \mathcal{K}} z_{ijk} = 1,$$

One route operation is scheduled only once

$$z_{ijk} \leq \lambda_{ijk},$$

Operation assigned to an allowed resource k

$$y_{ijpqk} + y_{pqijk} \leq z_{ijk},$$

$$y_{ijpqk} + y_{pqijk} + 1 \geq z_{ijk} + z_{pqk},$$

These two constraints regulates the ordering of the operations for a resource k

$$t_{ij} + p_{ij} - M(1 - y_{ijpqk}) \leq t_{pq},$$

Starting time (p,q) after compl. time (i,j) if same k

Big number

To be cont'd…

The optimization model cont'd

$$
\begin{aligned}
t_{ij} + p_{ij} + w &\leq t_{i+1,j}, && i \in \mathcal{N}_j \setminus \{n_j\}, \\
t_{1j} &\geq r_j, \\
t_{ij} &\geq a_k z_{ijk}, \\
t_{1q} &\geq s_j + v_{jq}, && (j,q) \in \mathcal{Q}, \\
s_j &= t_{n_j j} + p_{n_j j}, \\
h_j &\geq s_j - d_j, \\
h_j &\geq 0, \\
t_{ij} &\geq 0, \\
z_{ijk} &\in \{0,1\}, \\
y_{ijpqk} &\in \{0,1\}, && (i,j) \neq (p,q).
\end{aligned}
$$

The optimization model cont’d

$t_{ij} + p_{ij} + w \leq t_{i+1,j},$	Operation processed and transported before next
$t_{1j} \geq r_j,$	Job may be started after release date
$t_{ij} \geq a_k z_{ijk},$	Resource *k* available first time
$t_{1q} \geq s_j + v_{jq},$	Job *q* may be started after completion of job *j* + planned lead time
$s_j = t_{n_j j} + p_{n_j j},$	Definition of completion time
$h_j \geq s_j - d_j,$ $h_j \geq 0,$	Definition of tardiness
$t_{ij} \geq 0,$	Positive starting times
$z_{ijk} \in \{0, 1\},$ $y_{ijpqk} \in \{0, 1\},$	Binary variables

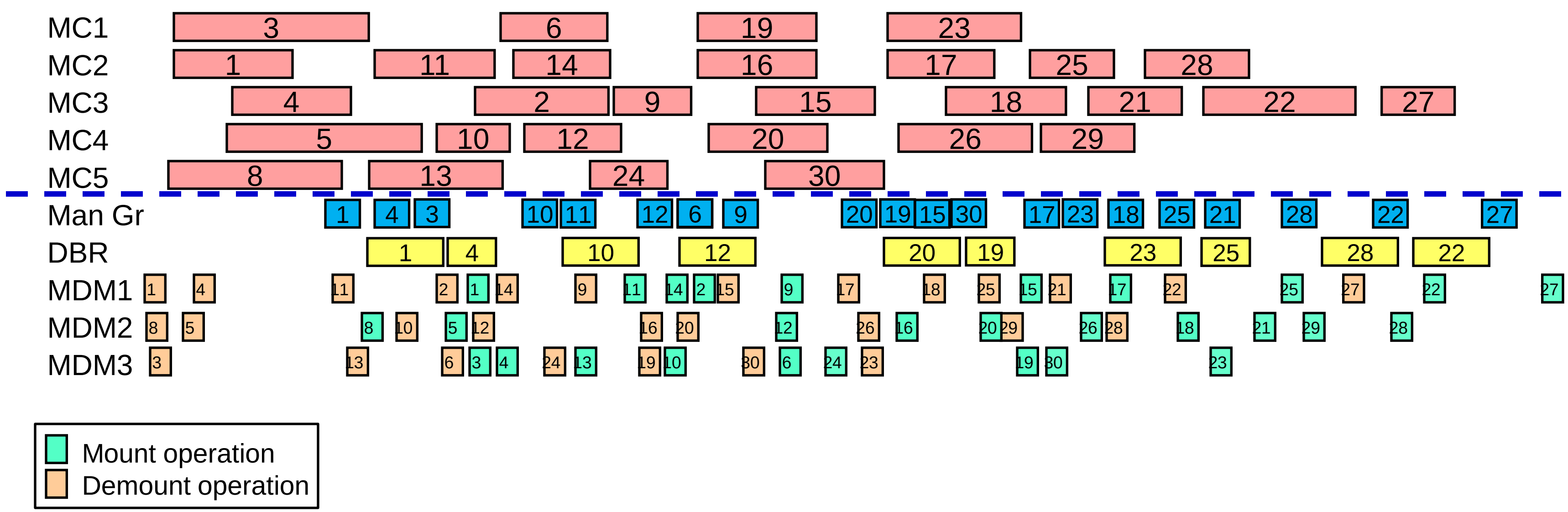
Division into two models

Too high CPU times for the whole model (AMPL-CPLEX11)

- The processing times of the machining resources >> other route operations
- Machining resources most heavy investments

The model divided into two models:

- The **machining model** optimizes the schedule of the machining resources MC1-5
- The **feasibility model** finds a feasible schedule for the rest of the route operations



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The machining problem

$$v_{jq}^{\mathrm{m}} = v_{jq} + t_{1q}$$

$$p_j^{\mathrm{pm}} = \sum_{i=3}^{n_j} p_{ij}$$

$$
\begin{array}{lrll}
\text{Minimize} & \sum_{j \in J} (s_j^{\mathrm{m}} + h_j^{\mathrm{m}}) & & \\
\text{subject to} & \sum_{k \in \mathcal{K}} z_{jk}^{\mathrm{m}} = 1, & & j \in \mathcal{J}, \\
& z_{jk}^{\mathrm{m}} \leq \lambda_{jk}^{\mathrm{m}}, & & j \in \mathcal{J},\ k \in \mathcal{K}, \\
& y_{jqk}^{\mathrm{m}} + y_{qjk}^{\mathrm{m}} \leq z_{jk}^{\mathrm{m}}, & & j,q \in \mathcal{J},\ k \in \mathcal{K},\ j \neq q, \\
& y_{jqk}^{\mathrm{m}} + y_{qjk}^{\mathrm{m}} + 1 \geq z_{jk}^{\mathrm{m}} + z_{qk}^{\mathrm{m}}, & & j,q \in \mathcal{J},\ k \in \mathcal{K},\ j \neq q, \\
& t_j^{\mathrm{m}} + p_j^{\mathrm{m}} - M(1 - y_{jqk}^{\mathrm{m}}) \leq t_q^{\mathrm{m}}, & & j,q \in \mathcal{J},\ k \in \mathcal{K},\ j \neq q, \\
& t_j^{\mathrm{m}} \geq r_j^{\mathrm{m}}, & & j \in \mathcal{J}, \\
& t_j^{\mathrm{m}} \geq a_k z_{jk}^{\mathrm{m}}, & & j \in \mathcal{J}, \\
& t_q^{\mathrm{m}} \geq s_j^{\mathrm{m}} + v_{jq}^{\mathrm{m}}, & & (j,q) \in \mathcal{Q}, \\
& s_j^{\mathrm{m}} = t_j^{\mathrm{m}} + p_j^{\mathrm{m}} + p_j^{\mathrm{pm}}, & & j \in \mathcal{J}, \\
& h_j^{\mathrm{m}} \geq s_j^{\mathrm{m}} - d_j^{\mathrm{m}}, & & j \in \mathcal{J}, \\
& h_j^{\mathrm{m}} \geq 0, & & j \in \mathcal{J}, \\
& t_j^{\mathrm{m}} \geq 0, & & j \in \mathcal{J}, \\
& z_{jk}^{\mathrm{m}} \in \{0,1\}, & & j \in \mathcal{J},\ k \in \mathcal{K}, \\
& y_{jqk}^{\mathrm{m}} \in \{0,1\}, & & j,q \in \mathcal{J},\ k \in \mathcal{K},\ j \neq q,
\end{array}
$$

The feasibility model

$$\text{Minimize} \sum_{j\in\mathcal{J}} \left(s_j - 0.001 t_{1j} + h_j + \sum_{i\in\mathcal{N}_j} \sum_{k\in\mathcal{K}} \omega_k z_{ijk} \right)$$

different weights for different setup stations

$$
\begin{aligned}
\text{subject to} \quad \sum_{k\in\mathcal{K}} z_{ijk} &= 1, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\\
z_{ijk} &\le \lambda_{ijk}, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\ k\in\mathcal{K},\\
y_{ijpqk} + y_{pqijk} &\le z_{ijk}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\ k\in\mathcal{K},\ (i,j)\neq(p,q),\\
y_{ijpqk} + y_{pqijk} + 1 &\ge z_{ijk} + z_{pqk}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\ k\in\mathcal{K},\ (i,j)\neq(p,q),\\
t_{ij} + p_{ij} - M(1-y_{ijpqk}) &\le t_{pq}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\ k\in\mathcal{K},\ (i,j)\neq(p,q),\\
t_{ij} + p_{ij} + w &\le t_{i+1,j}, && i\in\mathcal{N}_j\setminus\{n_j\},\ j\in\mathcal{J},\\
t_{1j} &\ge r_j, && j\in\mathcal{J},\\
t_{ij} &\ge a_k z_{ijk}, && j\in\mathcal{J},\ k\in\mathcal{K},\\
t_{1q} &\ge s_j + v_{jq}, && (j,q)\in\mathcal{Q},\\
s_j &= t_{n_j j} + p_{n_j j}, && j\in\mathcal{J},\\
h_j &\ge s_j - d_j, && j\in\mathcal{J},\\
h_j &\ge 0, && j\in\mathcal{J},\\
t_{ij} &\ge 0, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\\
y_{2j2qk} &= y^m_{jqk}, && j,q\in\mathcal{J},\ k\in\mathcal{K},\\
z_{2jk} &= z^m_{jk}, && j\in\mathcal{J},\ k\in\mathcal{K},\\
z_{ijk} &\in \{0,1\}, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\ k\in\mathcal{K},\\
y_{ijpqk} &\in \{0,1\}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\ k\in\mathcal{K},\ (i,j)\neq(p,q),
\end{aligned}
$$

Fixed to solution from machining problem

where y^m_{jqk} and z^m_{jk} are the solutions obtained from the machining problem.

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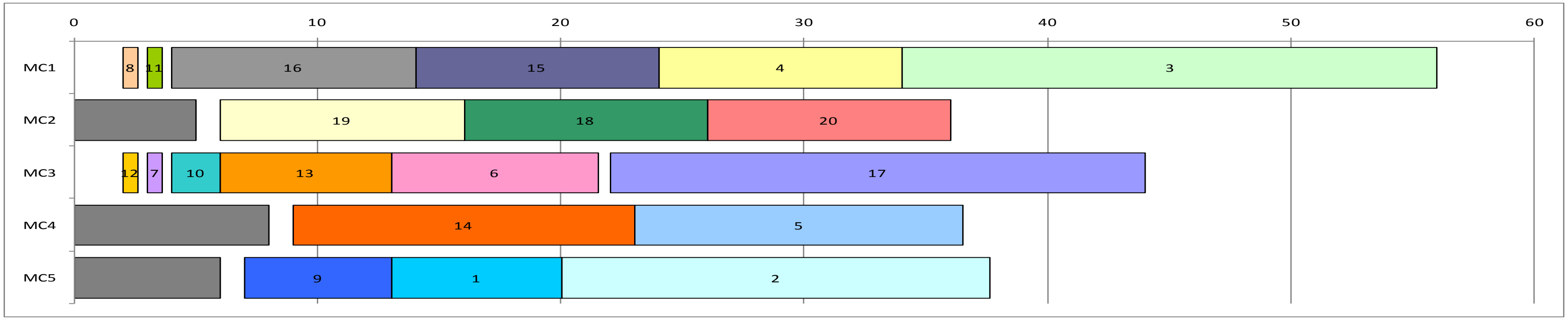
Discrete machining model



The time horizon of the schedule is divided into T+1 discrete time steps.

Variables:

$$x_{jkt} = \begin{cases} 1, & \text{if job } j \text{ is to start at the beginning of time interval } t \text{ at resource } k, \\ 0, & \text{otherwise.} \end{cases}$$

$$s_j = \text{the completion time of job } j.$$

$$h_j = \begin{cases} s_j + p_j^{\text{pm}} - d_j, & \text{if } s_j + p_j^{\text{pm}} > d_j, \text{ i.e. the tardiness of job } j, \\ 0, & \text{otherwise.} \end{cases}$$

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The discrete machining problem

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j\in\mathcal{J}} (s_j + h_j), \\
\text{subject to} \quad & \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{T}} x_{jku} = 1, && j\in\mathcal{J}, \\
& \sum_{u\in\mathcal{T}} x_{jku} \le \lambda_{jk}, && j\in\mathcal{J},\ k\in\mathcal{K}, \\
& x_{jku} + x_{q,k,u+\mu} \le 1, && j,q\in\mathcal{J}, j\neq q, k\in\mathcal{K}, u=0,\ldots,\mathtt{T}-(p_j-1), \mu=0,\ldots,p_j-1, \\
& \sum_{k\in\mathcal{K}} \Big(x_{jku} + \sum_{u_{\mathrm{ext}}=0}^{u+v_{jq}^{\mathrm{ext}}-1} x_{qku_{\mathrm{ext}}}\Big) \le 1, && (j,q)\in\mathcal{Q},\ u=0,\ldots,\mathtt{T}-(v_{jq}^{\mathrm{ext}}-1), \\
& x_{jku} = 0, && (j,q)\in\mathcal{Q},\ k\in\mathcal{K},\ u=\mathtt{T}-v_{jq}^{\mathrm{ext}},\ldots,\mathtt{T}, \\
& x_{jku} = 0, && j\in\mathcal{J},\ k\in\mathcal{K},\ u=0,1,\ldots,r_j, \\
& x_{jku} = 0, && j\in\mathcal{J},\ k\in\mathcal{K},\ u=0,1,\ldots,a_k, \\
& \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{T}} u x_{jku} + p_j + p_j^{\mathrm{pm}} = s_j, && j\in\mathcal{J}, \\
& h_j \ge s_j - d_j, && j\in\mathcal{J}, \\
& h_j \ge 0, && j\in\mathcal{J}, \\
& x_{jku} \in \{0,1\}, && j\in\mathcal{J},\ k\in\mathcal{K},\ u\in\mathcal{T},
\end{aligned}
$$

where $v_{jq}^{\mathrm{ext}} = p_j + p_j^{\mathrm{pm}} + v_{jq}$.

The discrete machining problem

$$\text{Minimize} \quad \sum_{j\in\mathcal{J}}(s_j + h_j),$$

Objective: Minimize the sum of completion times and tardiness.

$$\text{subject to} \quad \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{T}} x_{jku} = 1,$$

One job is scheduled only once

$$\sum_{u\in\mathcal{T}} x_{jku} \leq \lambda_{jk},$$

Each job can only be assigned to an allowed resource k

$$x_{jku} + x_{q,k,u+\mu} \leq 1,$$

Only one job at a time can be processed on resource k

$$\sum_{k\in\mathcal{K}}(x_{jku} + \sum_{u_{\text{ext}}=0}^{u+v_{jq}^{\text{ext}}-1} x_{qku_{\text{ext}}}) \leq 1,$$

Job *q* may be started after completion of job *j* + planned lead time between the jobs on the same part

$$x_{jku} = 0,$$

$$x_{jku} = 0, \quad j\in\mathcal{J},\ k\in\mathcal{K},\ u = 0, 1, \ldots, r_j,$$

Release date

$$x_{jku} = 0, \quad j\in\mathcal{J},\ k\in\mathcal{K},\ u = 0, 1, \ldots, a_k,$$

Resource availability

$$\sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{T}} u x_{jku} + p_j + p_j^{\text{pm}} = s_j,$$

Definition of completion time

$$h_j \geq s_j - d_j$$

Definition of tardiness

$$h_j \geq 0,$$

$$x_{jku} \in \{0, 1\},$$

Binary variables

$$\text{where } v_{jq}^{\text{ext}} = p_j + p_j^{\text{pm}} + v_{jq}.$$

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Comparison

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j\in\mathcal{J}} (s_j + h_j), \\
\text{subject to} \quad & \sum_{k\in\mathcal{K}} \sum_{u\in\mathcal{T}} x_{jku} = 1, \\
& \sum_{u\in\mathcal{T}} x_{jku} \le \lambda_{jk}, \\
& x_{jku} + x_{q,k,u+\mu} \le 1, \\
& \sum_{k\in\mathcal{K}} \Big(x_{jku} + \sum_{u_{\text{ext}}=0}^{u+v^{\text{ext}}_{jq}-1} x_{qku_{\text{ext}}} \Big) \le 1, \\
& x_{jku} = 0, \\
& x_{jku} = 0, \\
& x_{jku} = 0, \\
& \sum_{k\in\mathcal{K}} \sum_{u\in\mathcal{T}} u x_{jku} + p_j + p_j^{\text{pm}} = s_j, \\
& h_j \ge s_j - d_j, \\
& h_j \ge 0, \\
& x_{jku} \in \{0, 1\},
\end{aligned}
$$

where $v^{\text{ext}}_{jq} = p_j + p_j^{\text{pm}} + v_{jq}$.

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j\in J} (s_j^m + h_j^m) \\
\text{subject to} \quad & \sum_{k\in\mathcal{K}} z_{jk}^m = 1, \\
& z_{jk}^m \le \lambda_{jk}^m, \\
& y_{jqk}^m + y_{qjk}^m \le z_{jk}^m, \\
& y_{jqk}^m + y_{qjk}^m + 1 \ge z_{jk} + z_{qk}, \\
& t_j^m + p_j^m - M(1 - y_{jqk}^m) \le t_q^m, \\
& t_j^m \ge r_j^m, \\
& t_j^m \ge a_k z_{jk}^m, \\
& t_q^m \ge s_j^m + v_{jq}, \\
& s_j^m = t_j^m + p_j^m + p_j^{pm}, \\
& h_j^m \ge s_j^m - d_j^m.
\end{aligned}
$$

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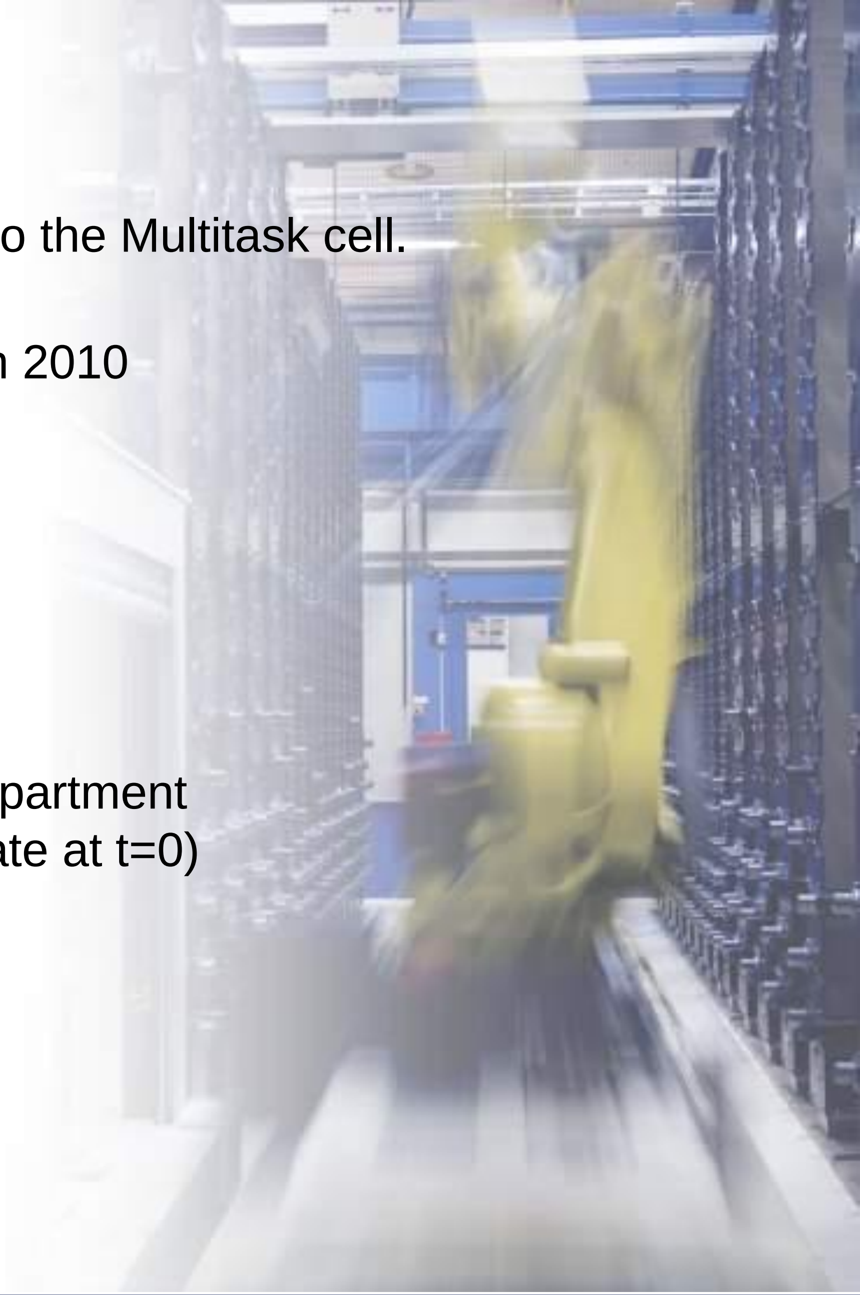
Test scenarios

All 20 jobs are assumed to be checked-in into the Multitask cell.

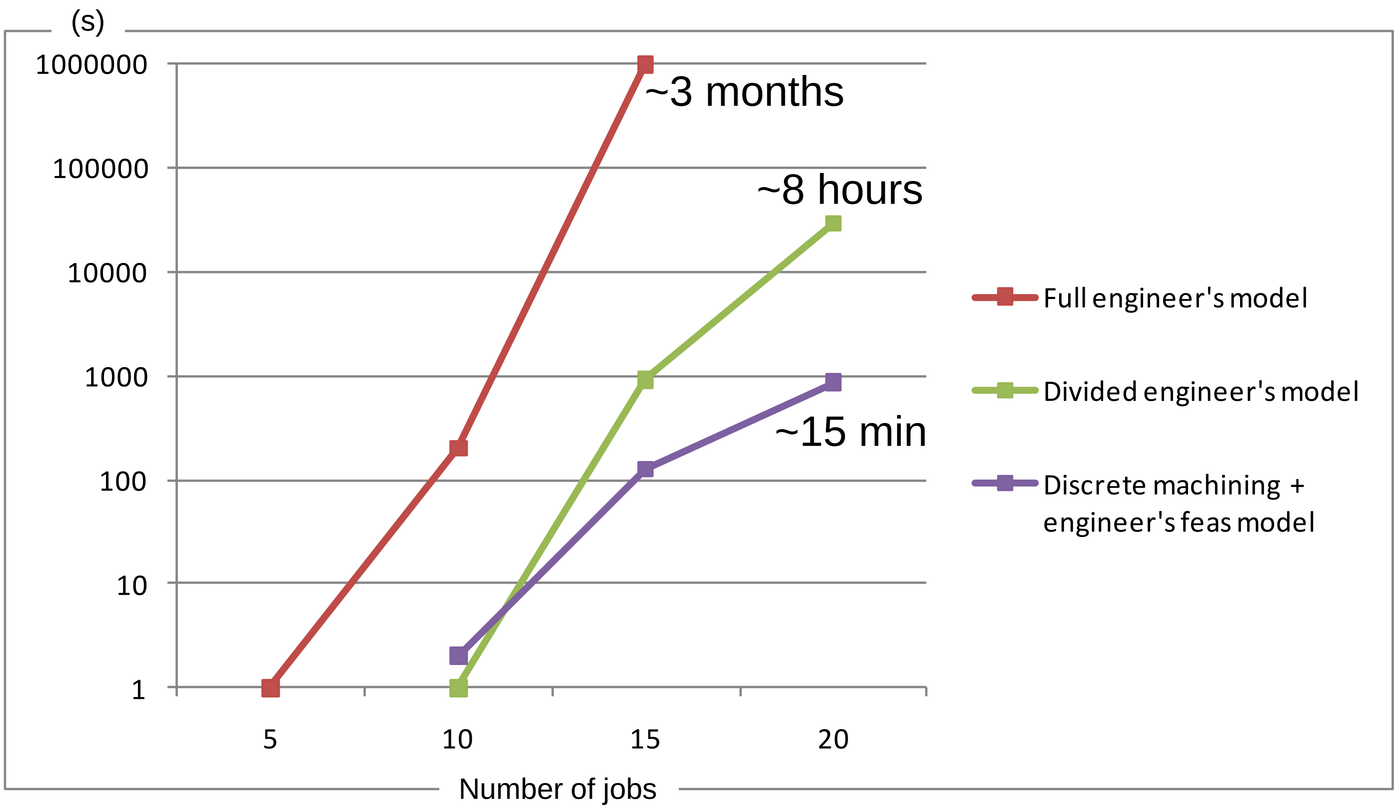
- Real production case – one day in March 2010
  (20 jobs, all jobs late at t=0)
    1. As is
    2. short jobs (25 %)
    3. long jobs (25 %)

- High volume case –
  created from prognosis by the market department
  (20 jobs, approximately half of the jobs late at t=0)
    1. As is
    2. short jobs (25 %)
    3. long jobs (25 %)



Computational results – CPU times

All computations have been carried out on a
4 Gb quad-core Intel Xeon 3.2 GHz system using AMPL-CPLEX12
Comparison of CPU times (seconds)



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Computational results

Results given as a mean per job, and the differences are relative to the completion time of the optimal solution.

Scenario	Scheduling algorithm	Completion time (h)	Diff from optimal solution (h)	Completion time diff (%)	Tardiness diff (h)	Tardiness diff (%)
Real	OPT	22.9	0	**0%**	0	**0%**
prod	FIFO	26.9	4.0	**18.0%**	4.0	**18.0%**
case	EDD	26.3	2.4	**10.4%**	2.4	**10.4%**
High	OPT	25.4	0	**0%**	0	**0%**
volume	FIFO	33.9	8.5	**33.9%**	5.7	**22.4%**
case	EDD	32.5	7.1	**28.9%**	2.9	**11.7%**

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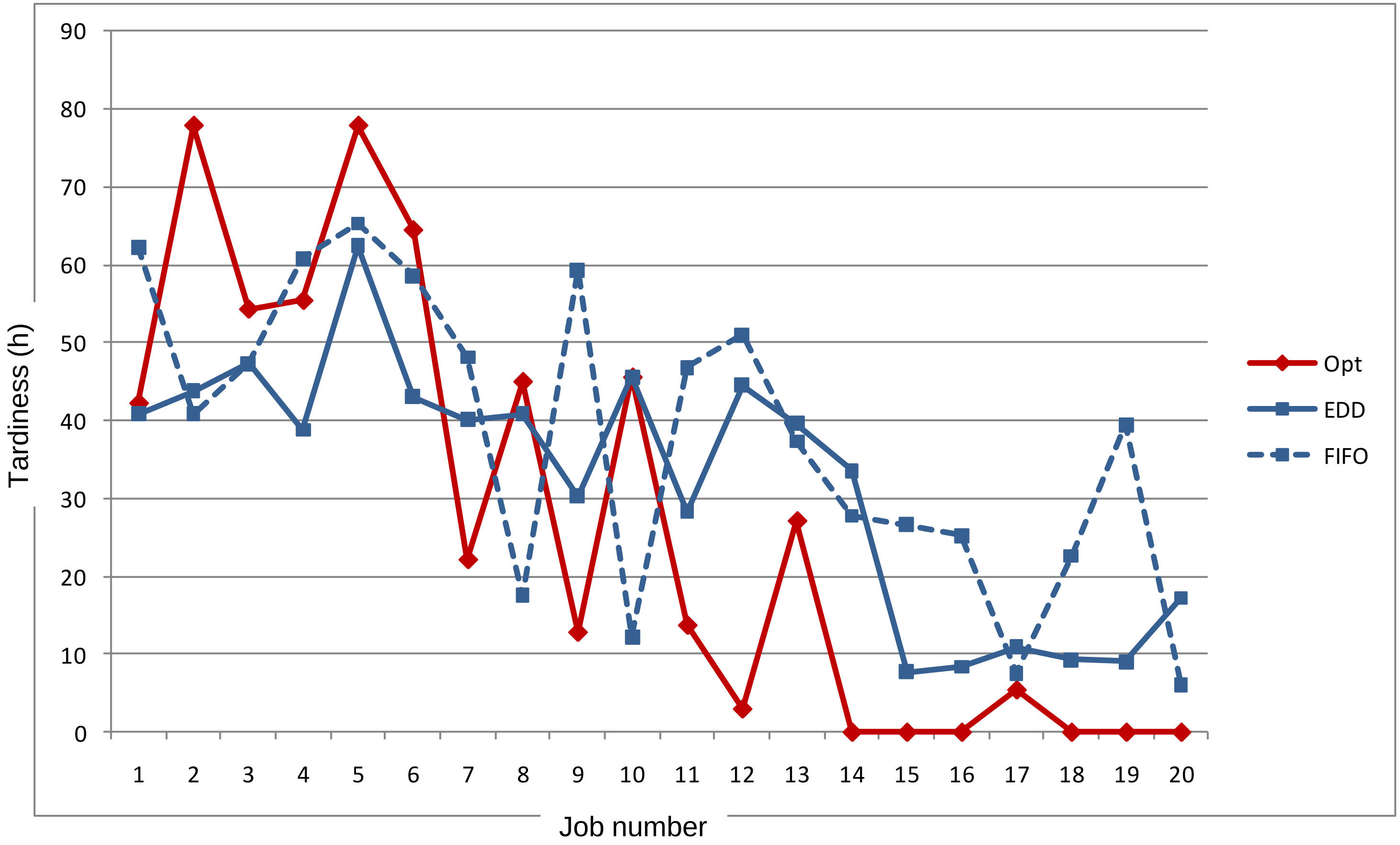
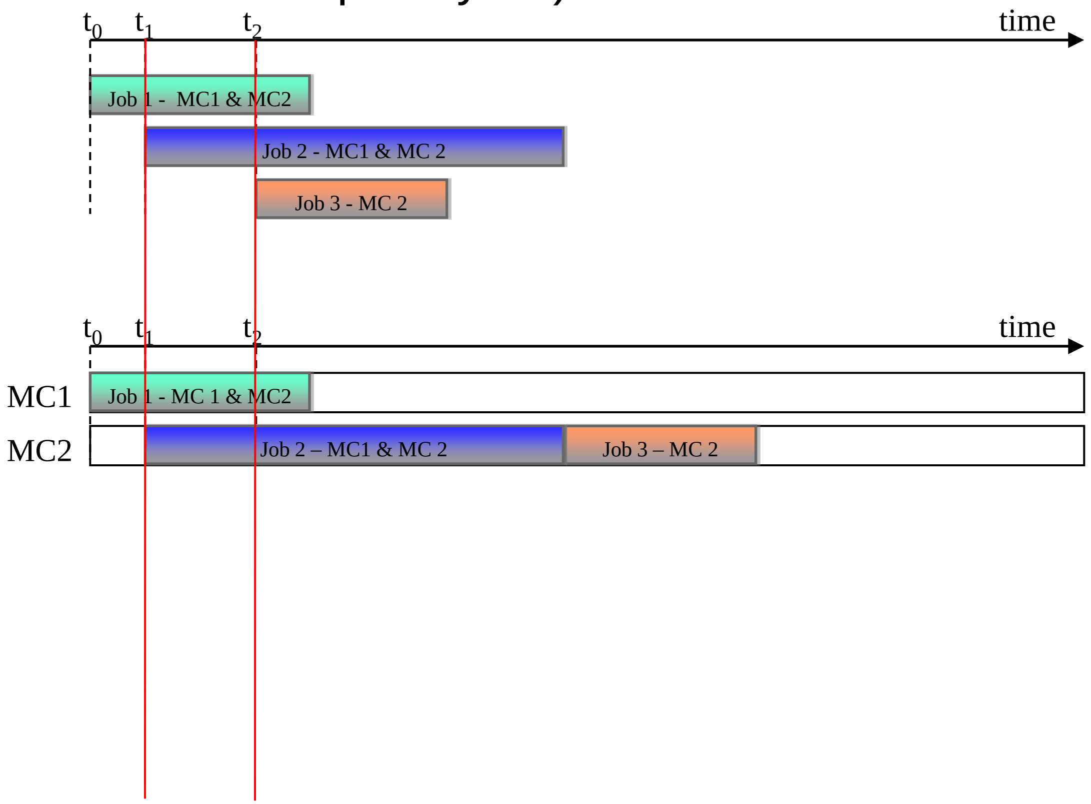
Job tardiness results

Tardiness results from high volume long jobs scenario



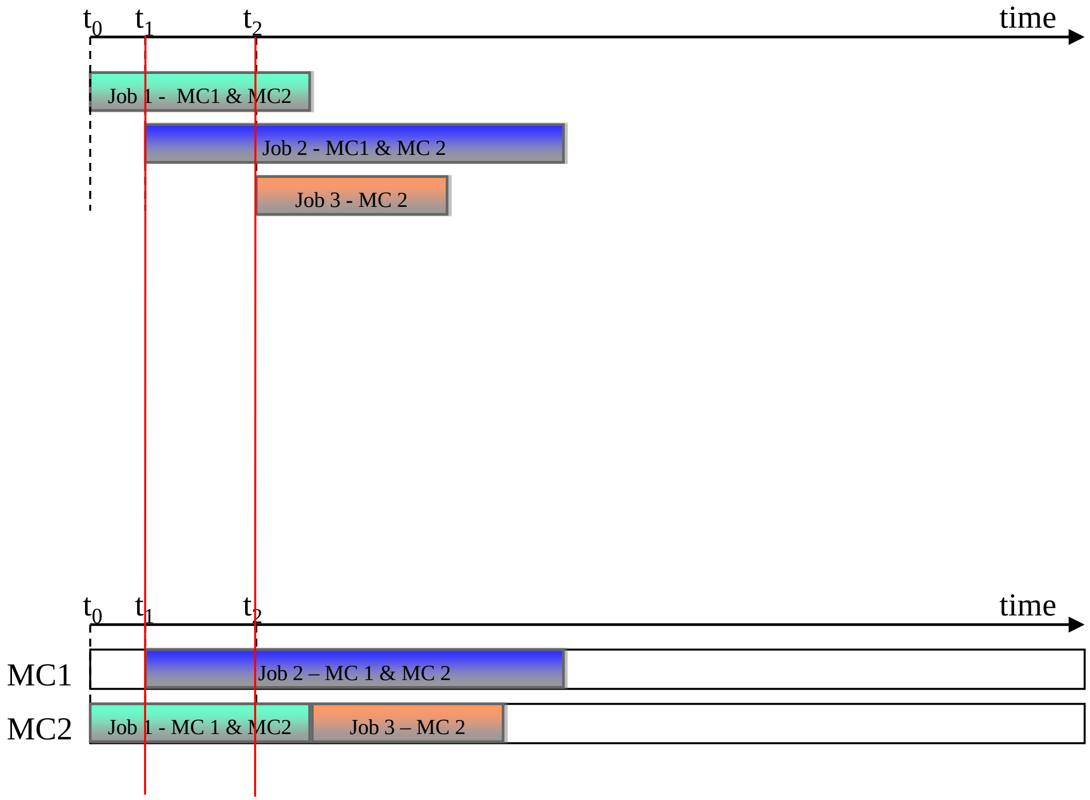
Shortsighted scheduling

No knowledge about which jobs are on the way to the multitask cell (or further down in the priority list)

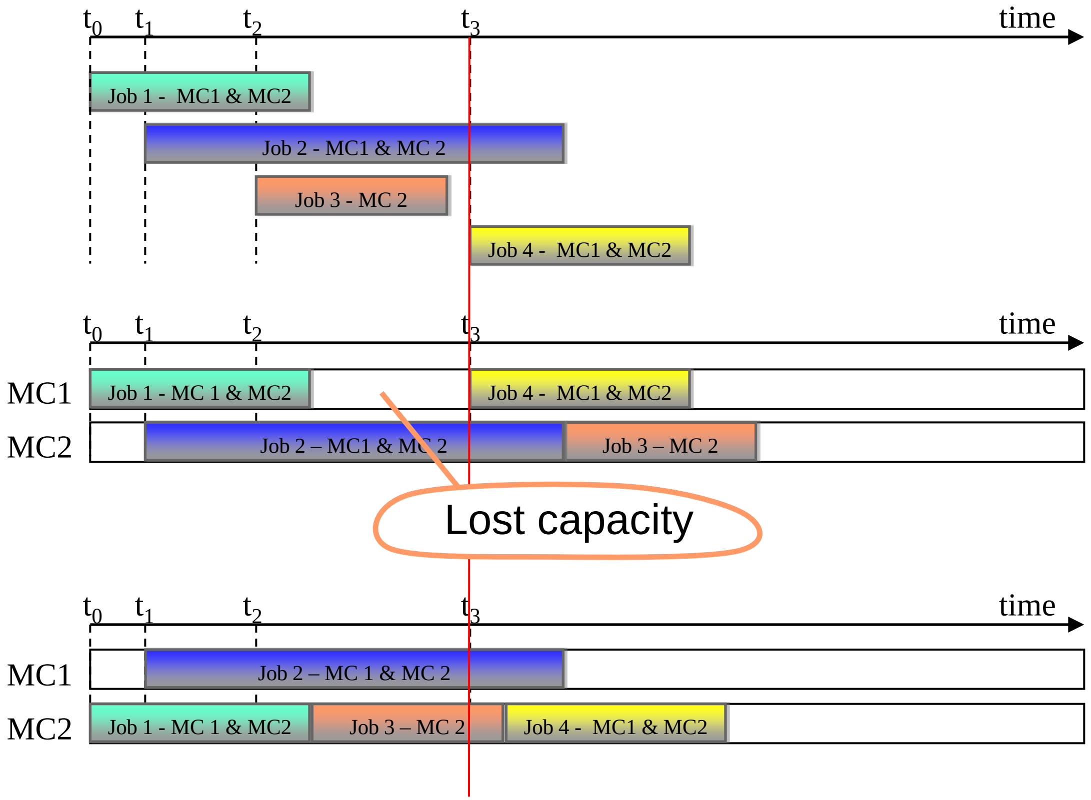


Looking into the future…

The optimization model takes all jobs in the queue into account



Looking into the future…



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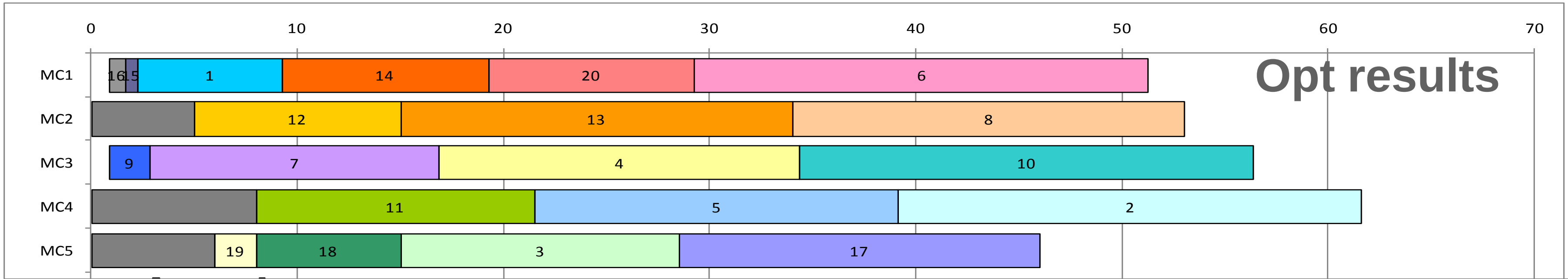
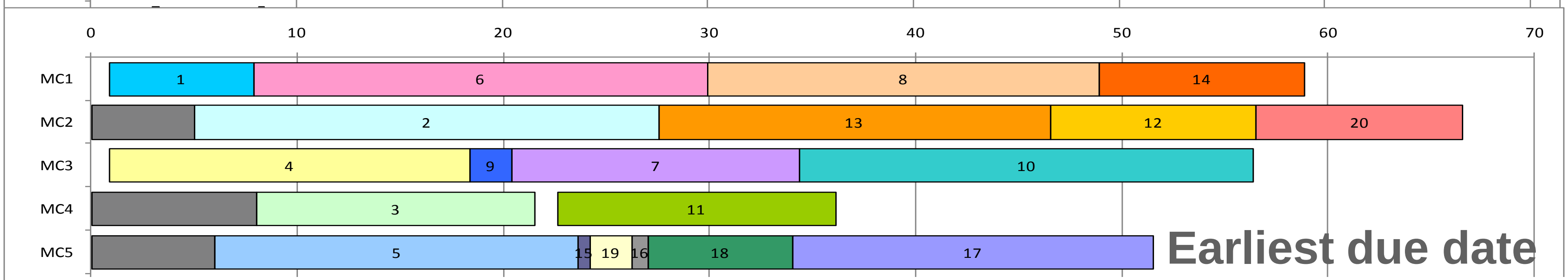
Scenario: high volume long jobs





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Coping with reality

As soon as the production schedule is optimized – something changes!

Expected events

- New parts in the queue
- Variances in the planned lead time

Unexpected events

- Machine breakdown
- Operator sick
- Part with non-conformance leaves queue
- etc.

Frequency:	when necessary
CPU time	< 15 min

Continued research

- Compute results on a broader spectra of scenarios – based both on realistic data and high volume cases
- Compare results to more sophisticated scheduling algorithms
- Evaluate the results with an existing simulation model

More tests

- Constraint programming
- Lagrangian relaxation to get better lower bounds
- Column generation
- Development of heuristics

More theory

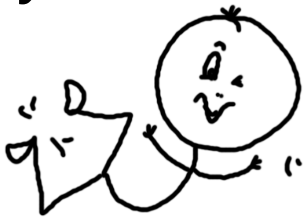
- More realistic model: fixtures, manpower etc.
- Find better objective functions

Better model

- …

Questions and comments?

Thank you



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Financial support from Volvo Aero, The Swedish Research Council, NFFP (Swedish National Aeronautics Research Programme)