Optimization of Schedules of a Multitask Production Cell

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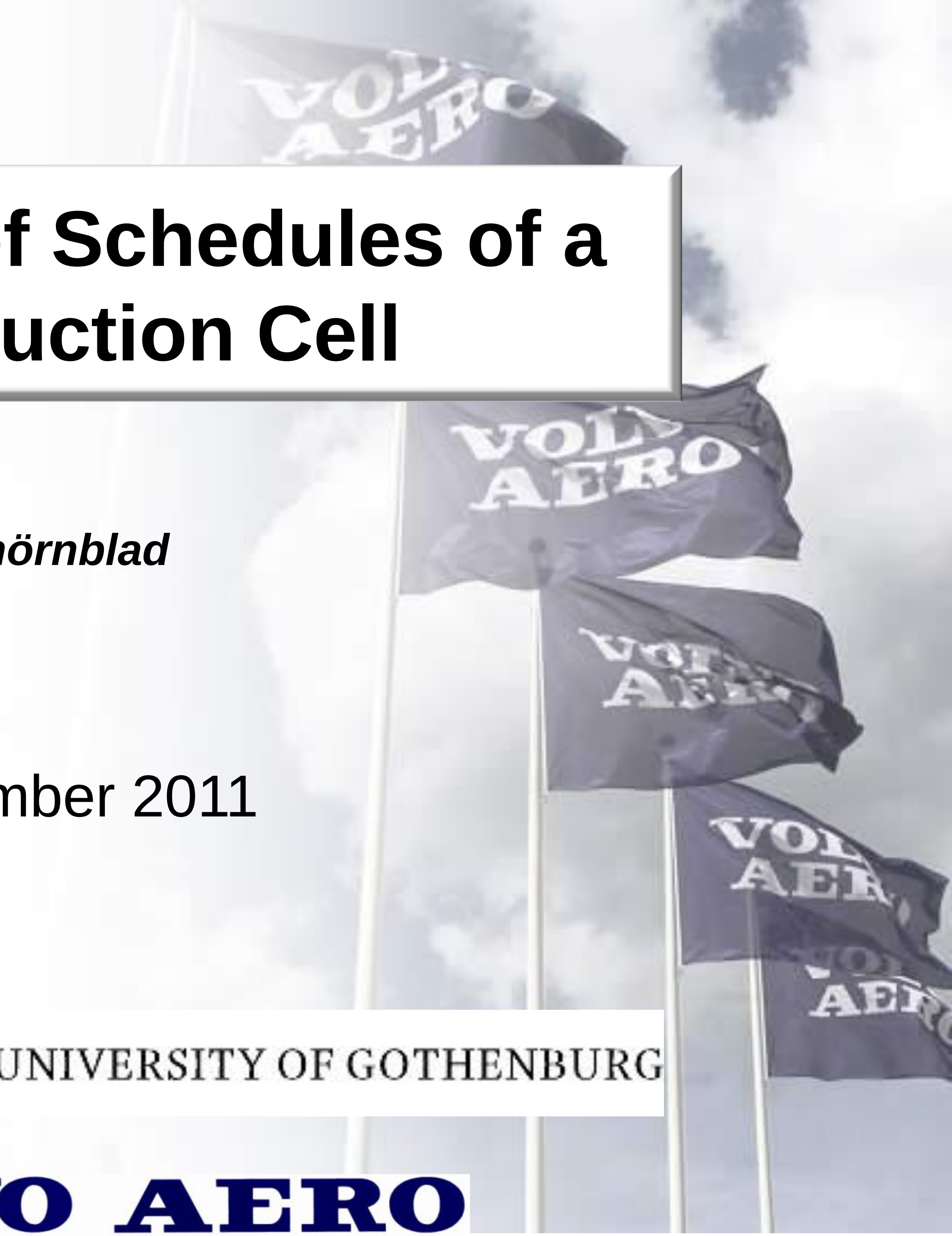


CHALMERS

UNIVERSITY OF GOTHENBURG

VOLVO AERO

VOLVO AERO

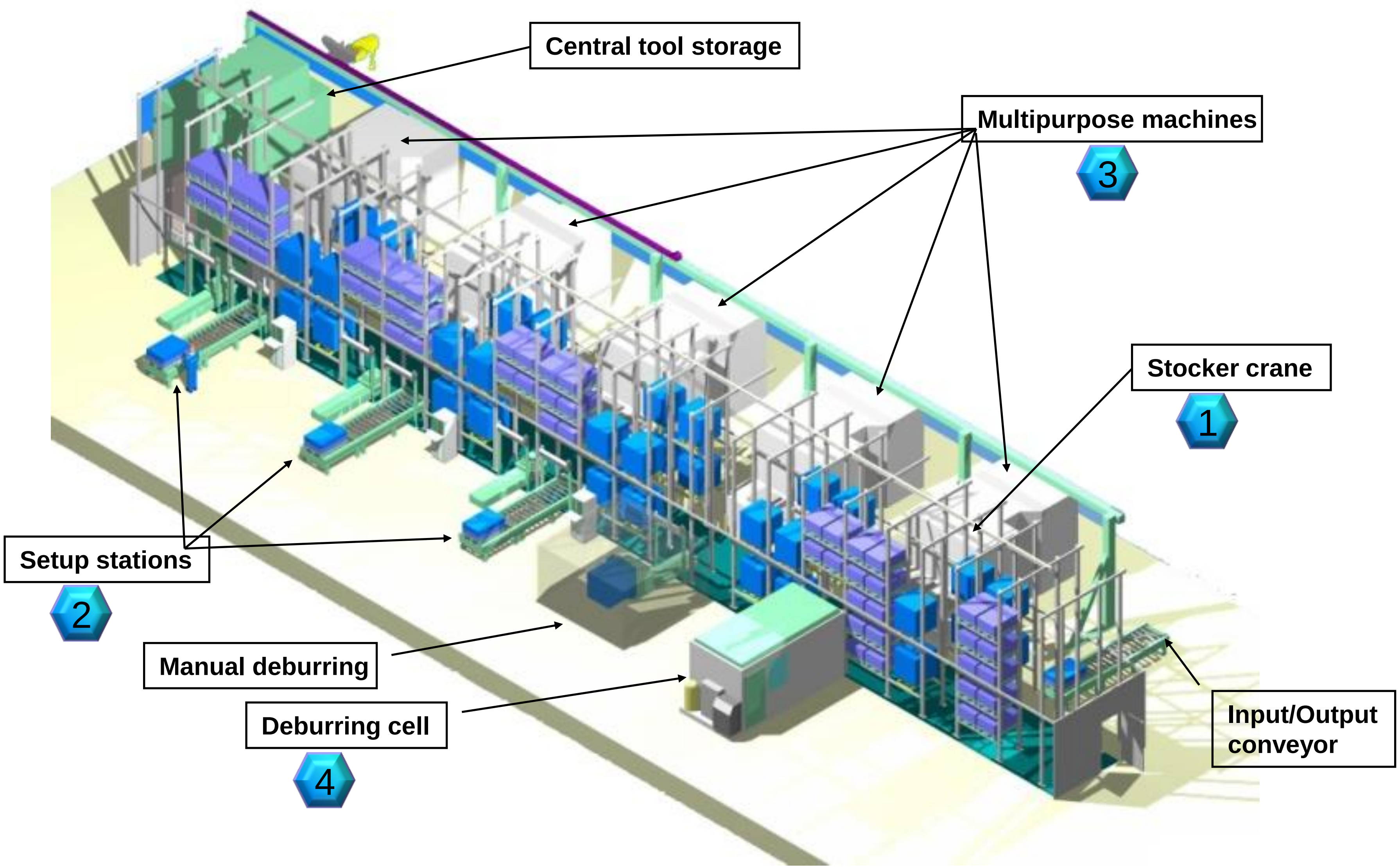
- Part of Volvo Group
- Develops and produces aircraft and rocket engine components
- About 3000 employees

Parts processed in the Multi Task Cell

~ 8 compressor rear frames for different aero engines and gas turbines.

About 30 different jobs are processed in the Multi Task Cell.

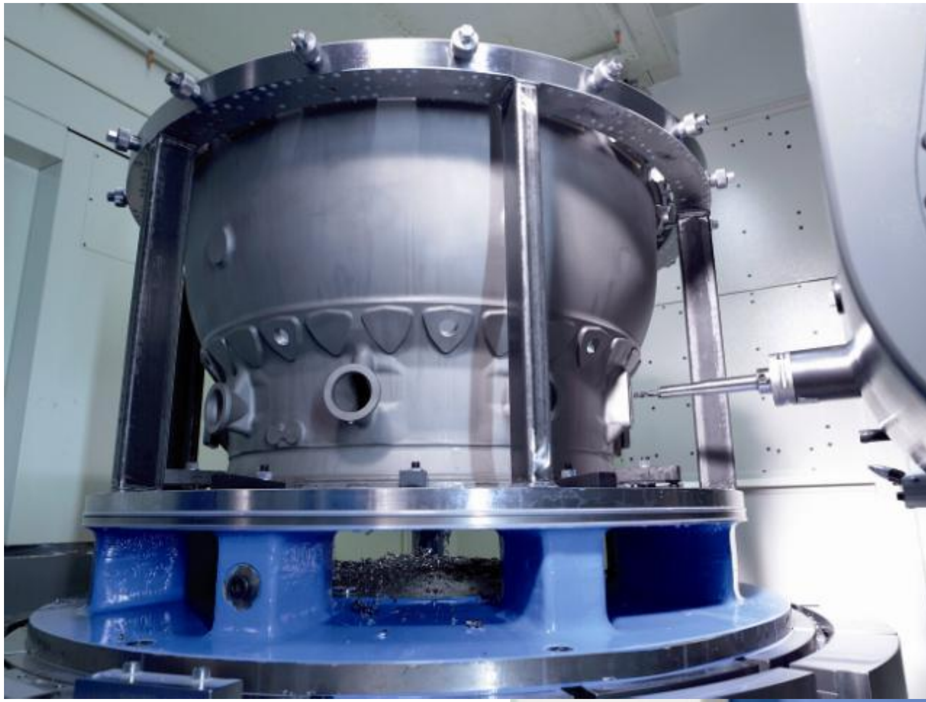
The Multitask Production Cell

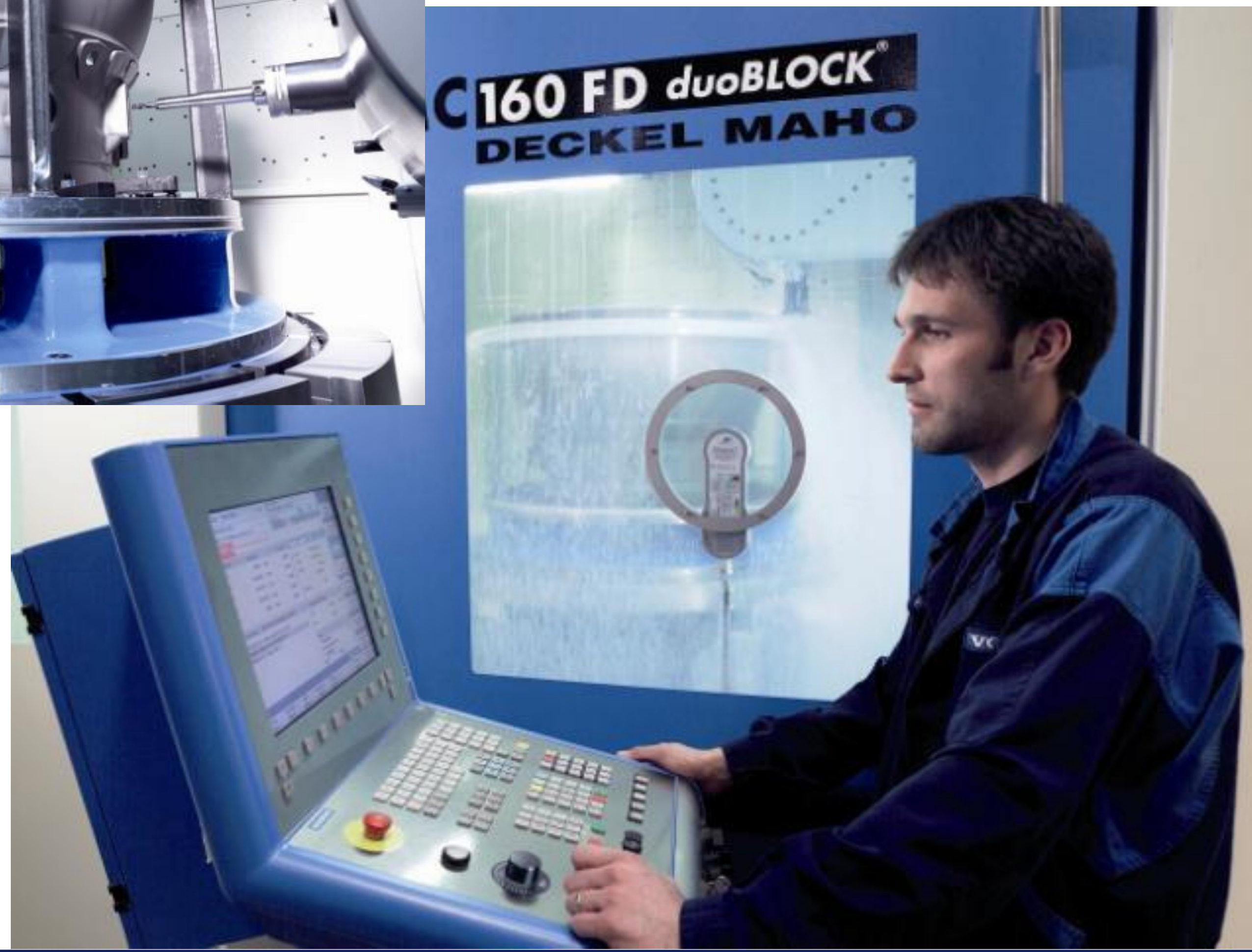


Stocker crane
Transports between storage and route operations

3 setup stations
Mount/demount in and out of fixtures

5 multitask machines
Drilling, milling and turning

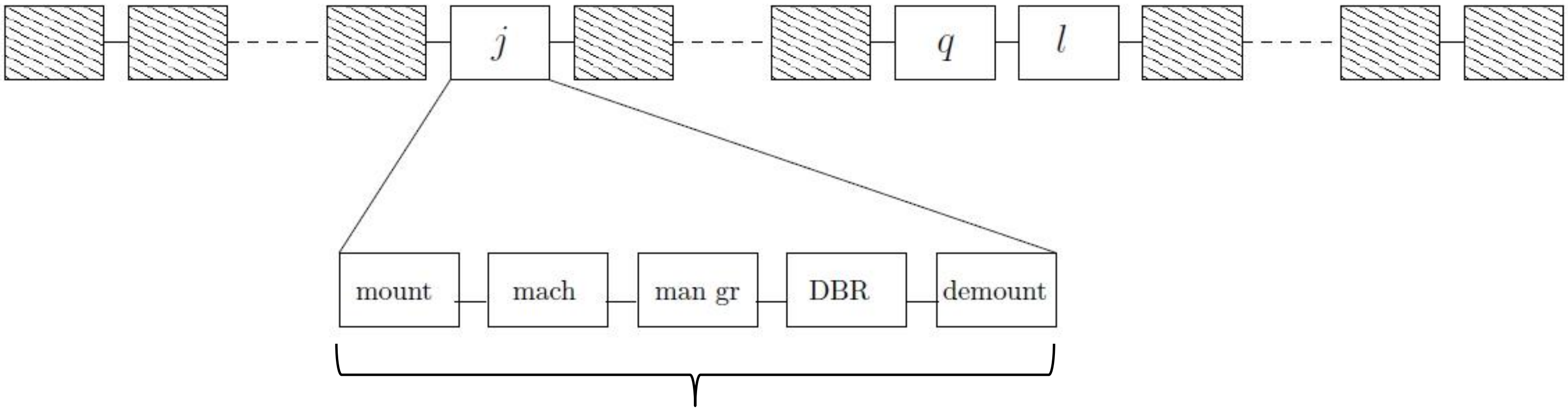




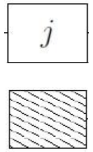
Automatic deburring cell
Robot deburring

The routing of a part

Every production order follows a routing in the planning system



One **job** in the multitask cell ↔ 3-5 route operations



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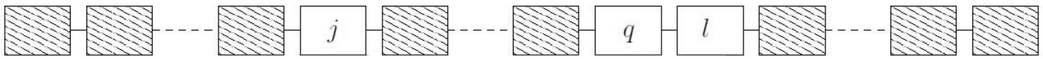
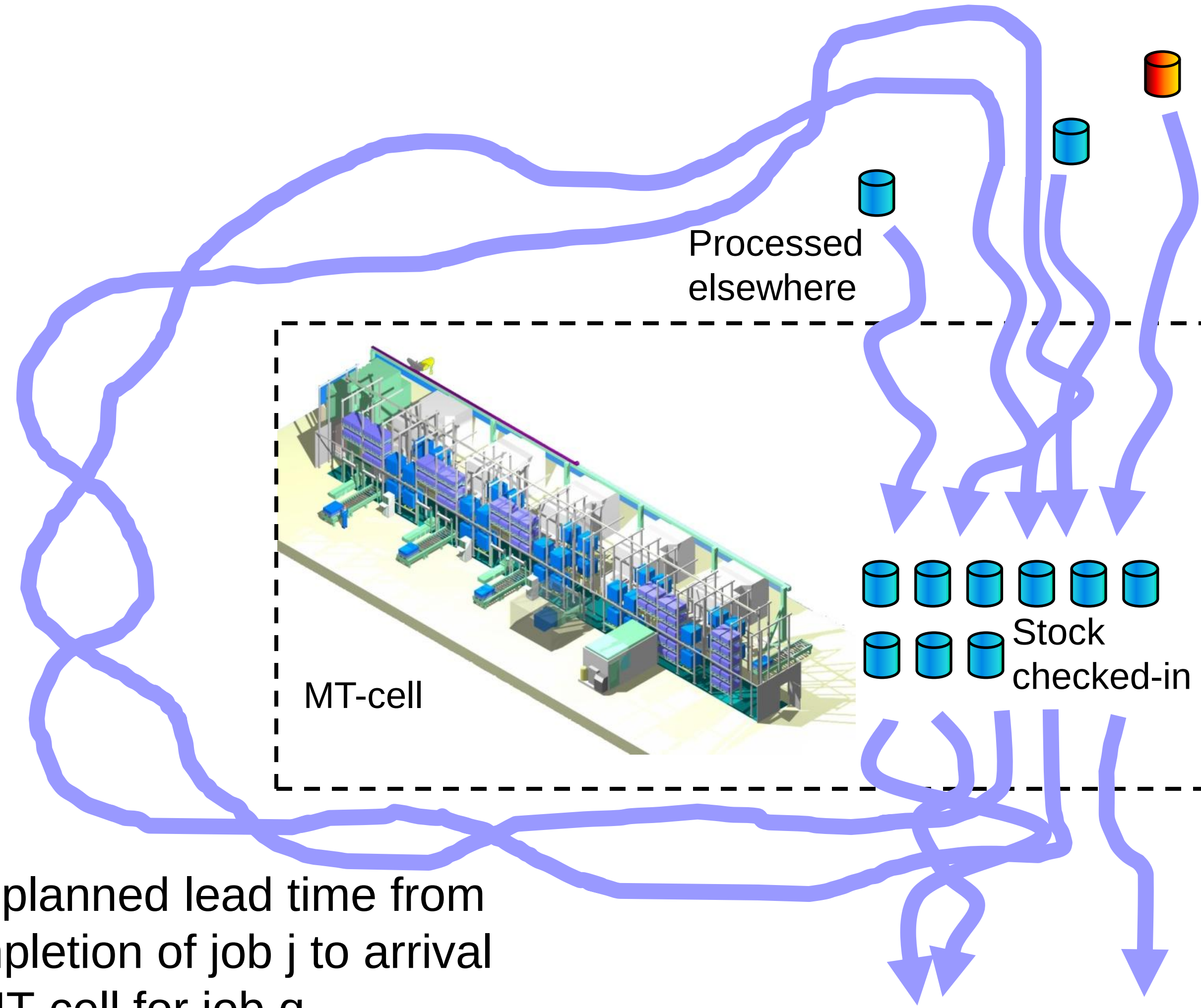
The queue of parts

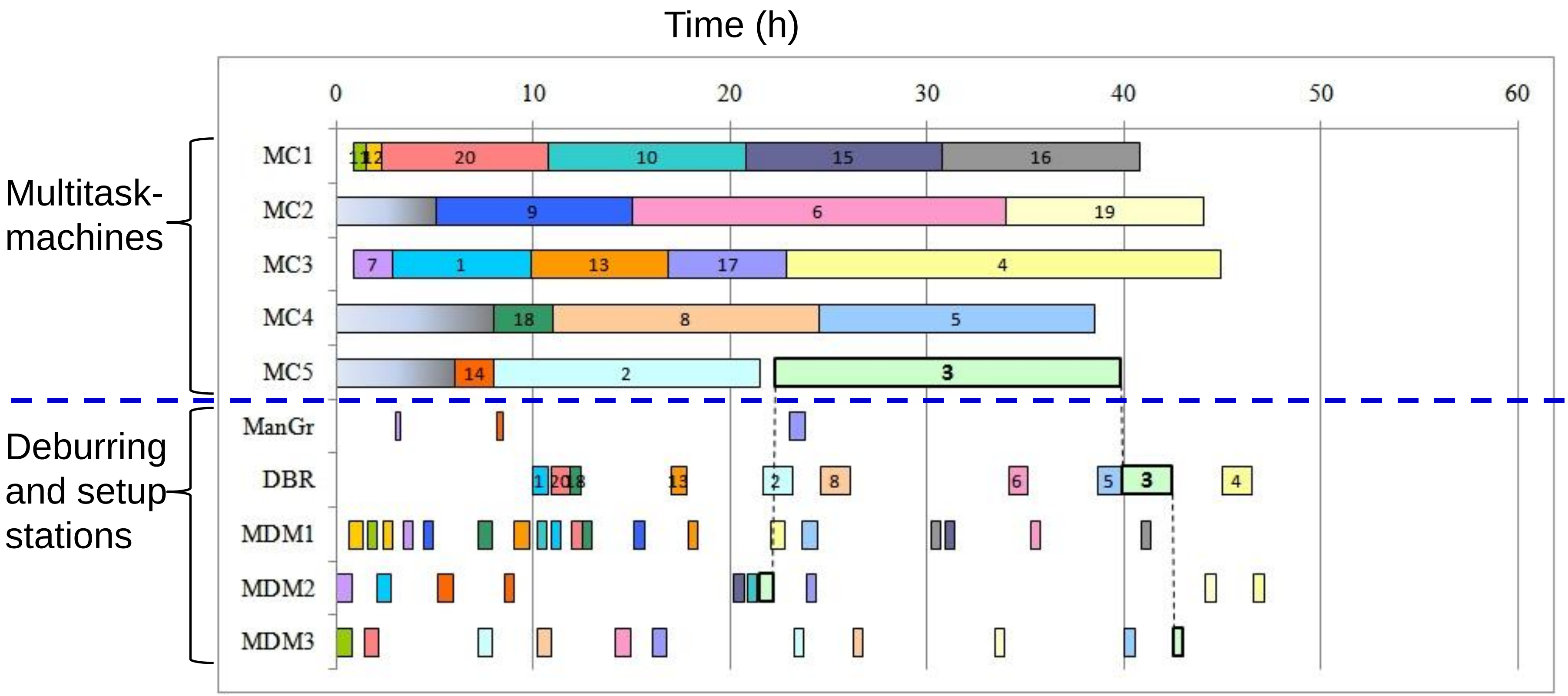




v_{0j}, planned lead time from current position to arrival at MT-cell

v_{jq} , planned lead time from completion of job j to arrival at MT-cell for job q

Problem decomposition



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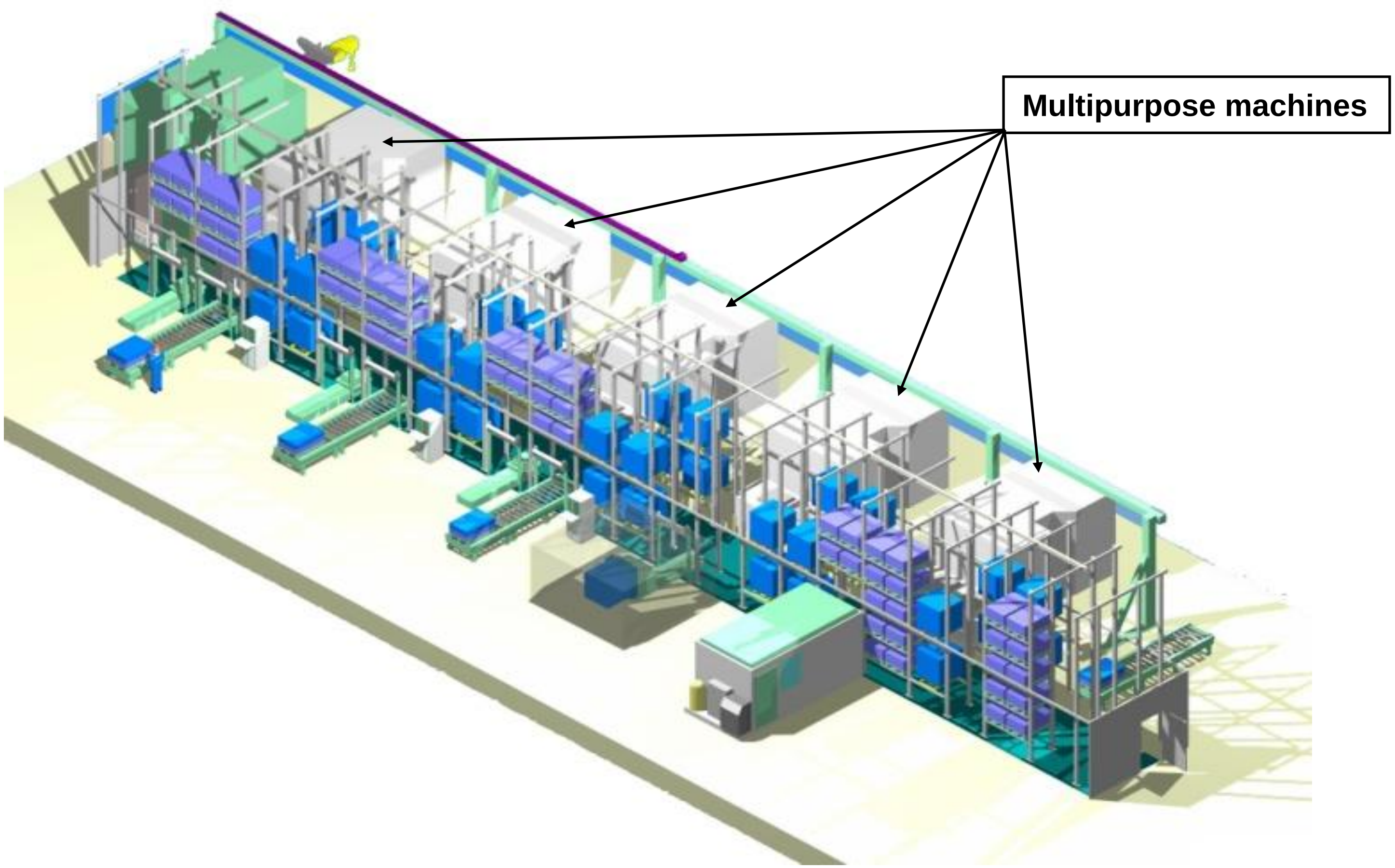
Notation

$$k \in \tilde{K}, \text{ set of resources}$$

$$\tilde{a}_k, \text{ first time when resource } k \text{ is available}$$



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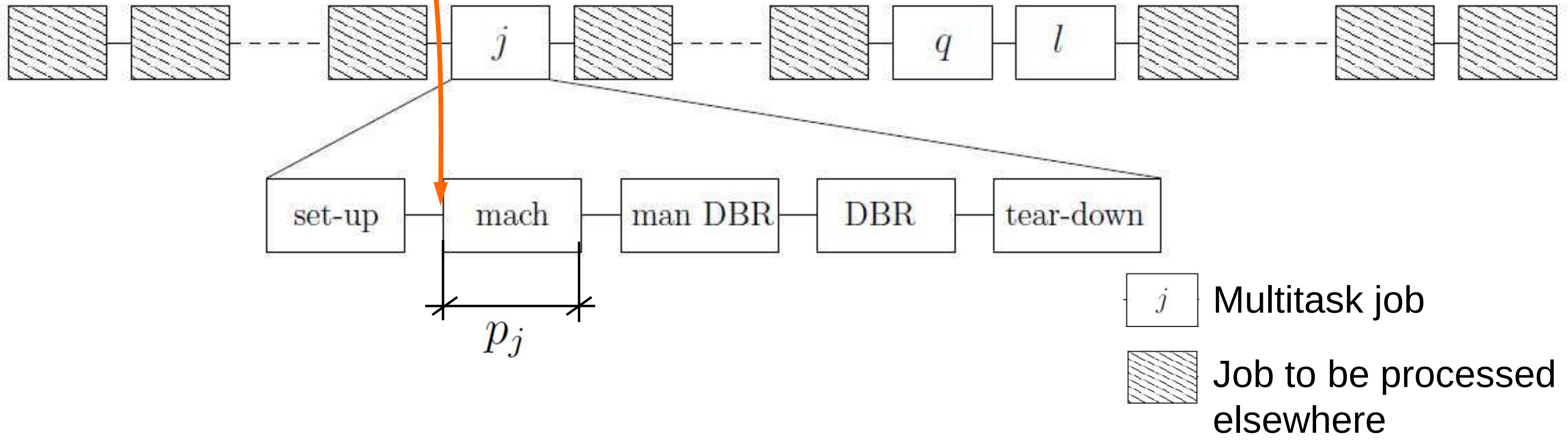
Notation

$$j \in \mathcal{J}, \text{ set of jobs}$$

$$\tilde{r}_j^{\mathtt{m}}, \text{ release dates}$$

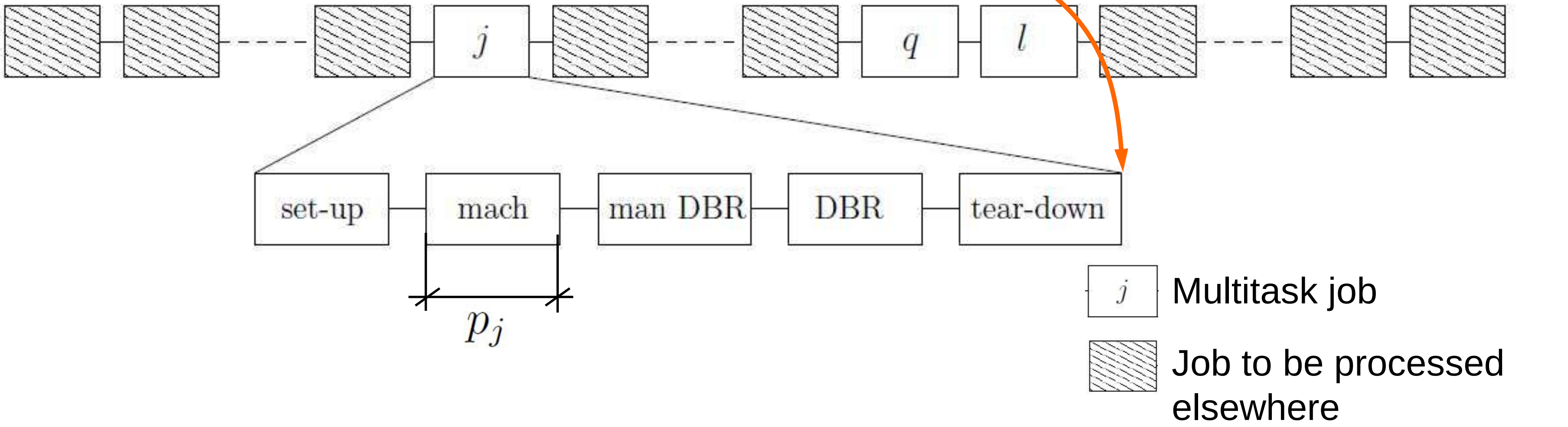


Notation

$$j \in \mathcal{J}, \text{ set of jobs}$$

$\tilde{r}_j^{\mathtt{m}}$, release dates

$\tilde{d}_j$, due dates



Notation

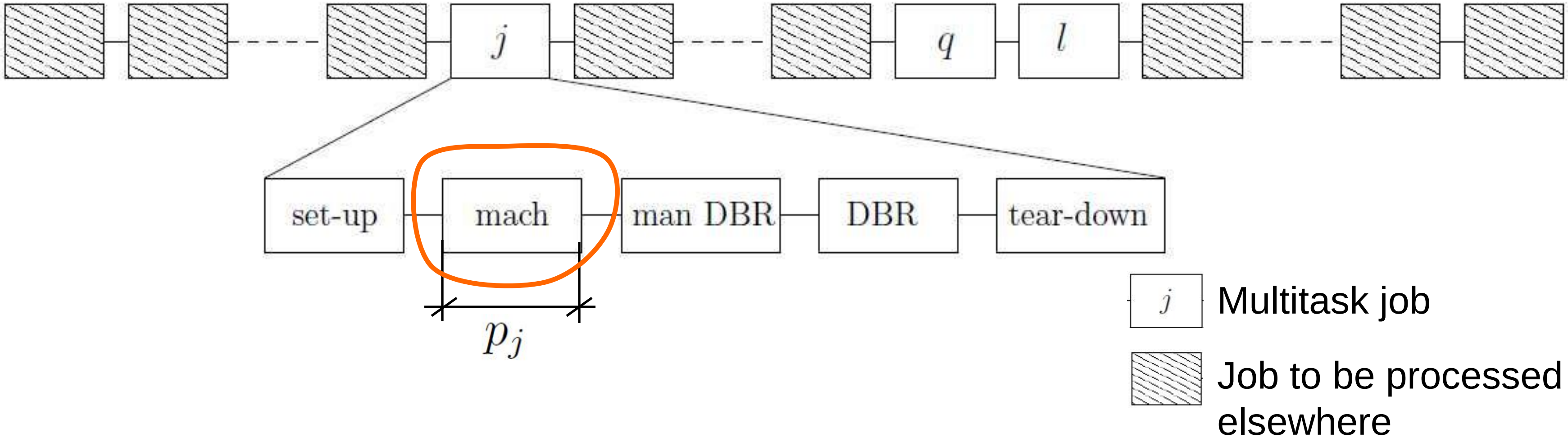
$$j \in \mathcal{J}, \text{ set of jobs}$$

$\tilde{r}_j^{\mathtt{m}}$, release dates

$\tilde{d}_j$, due dates

$$\lambda_{jk} = \begin{cases} 1, & \text{if job } j \text{ can be processed on resource } k, \\ 0, & \text{otherwise.} \end{cases}$$

$\tilde{p}_j$ machining processing time of job j.



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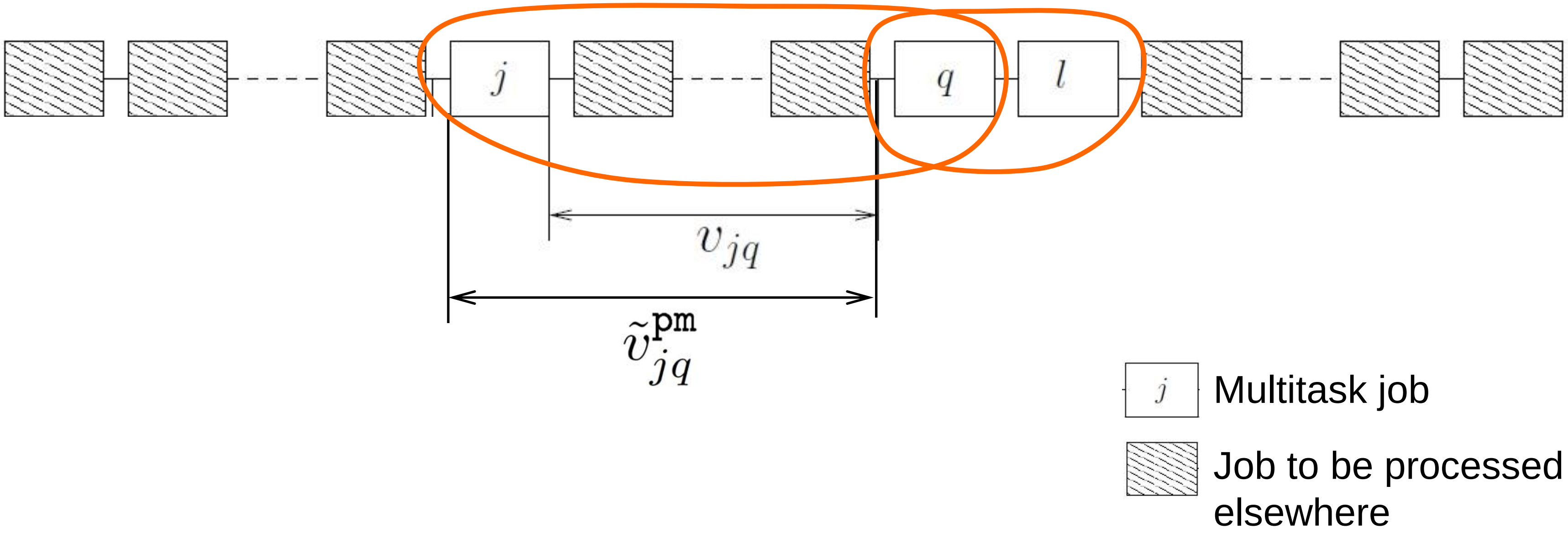
Notation

$(j, q) \in \mathcal{Q}$, set of pairs of subsequent jobs for the same physical part,

$\tilde{v}^{\text{pm}}_{jq}$ planned lead time between job j and job q



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Time-indexed formulation

ℓ, length of the discrete time interval

time interval u starts at time $u\ell$ hours

Decision variables

$$x_{jku} = \begin{cases} 1, & \text{if job } j \text{ is scheduled on } k \text{ at time } u \\ 0, & \text{otherwise.} \end{cases}$$

Binaries: $|\mathcal{J}||\mathcal{K}||\mathcal{T}|$ Test case No 3 (20 jobs): 9300

Time variables

$s_j = t_j + \tilde{p}_j + \tilde{p}_j^{\mathtt{pm}}$ the completion time of job j

$h_j = \max\{s_j - \tilde{d}_j; 0\}$, i.e. the tardiness of job j

Time–indexed formulation with nail variables

$$\text{Minimize} \quad \sum_{j\in\mathcal{J}}(As_j + Bh_j),$$

$$\text{subject to} \quad \sum_{k\in\tilde{\mathcal{K}}}\sum_{u\in\mathcal{T}} x_{jku} = 1, \qquad j\in\mathcal{J},$$

$$\sum_{u\in\mathcal{T}} x_{jku} \le \lambda_{jk}, \qquad j\in\mathcal{J}, k\in\tilde{\mathcal{K}},$$

$$\sum_{j\in\mathcal{J}}\sum_{\nu=(u-\tilde{p}_j+1)_+}^{u} x_{jk\nu} \le 1, \qquad k\in\tilde{\mathcal{K}}, u=0,\dots,T,$$

$$\boxed{\begin{aligned} &\sum_{k\in\tilde{\mathcal{K}}}\left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}_{jq}^{\mathrm{pm}}} x_{qk\nu}\right) \ge 0, && (j,q)\in\mathcal{Q}, u=0,\dots,T-\tilde{v}_{jq}^{\mathrm{pm}},\\ &x_{jku} = 0, && (j,q)\in\mathcal{Q},\ k\in\tilde{\mathcal{K}},\ u=T-\tilde{v}_{jq}^{\mathrm{pm}},\dots,T,\end{aligned}}$$

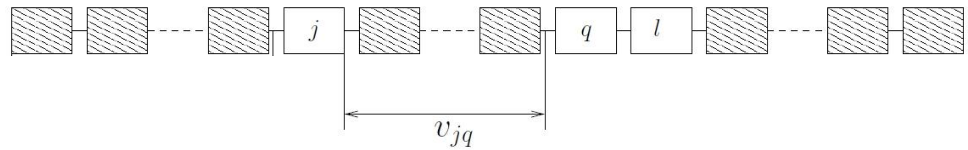
$$\sum_{k\in\tilde{\mathcal{K}}}\sum_{u\in\mathcal{T}} u x_{jku} + \tilde{p}_j^{\mathrm{pm}} = s_j, \qquad j\in\mathcal{J},$$

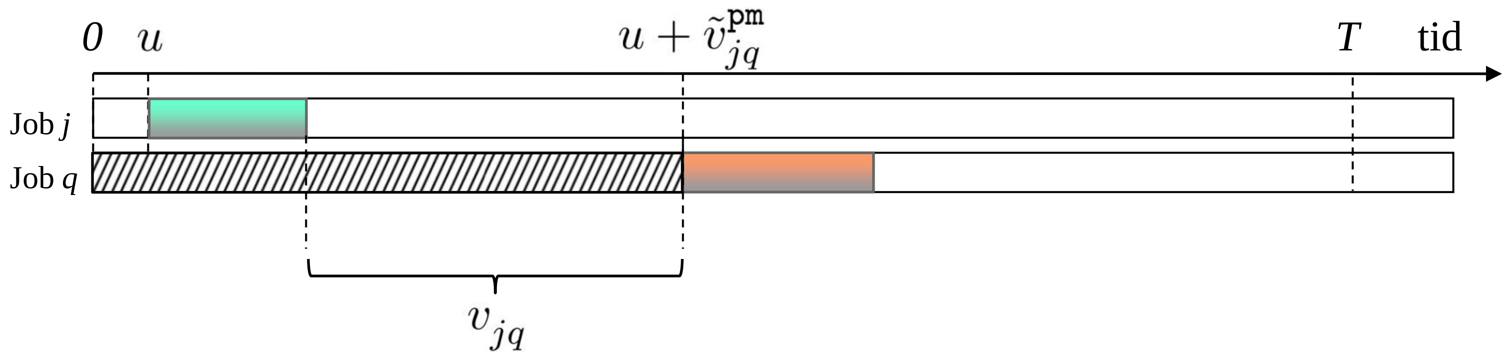
$$h_j \ge \max\{s_j - \tilde{d}_j; 0\}, \quad j\in\mathcal{J},$$

$$x_{jku} = 0, \qquad j\in\mathcal{J},\ k\in\tilde{\mathcal{K}},\ u=0,\dots,\max\{\tilde{r}_j^{\mathrm{m}}; \tilde{a}_k\},$$

$$x_{jku}\in\{0,1\}, \qquad j\in\mathcal{J},\ k\in\tilde{\mathcal{K}},\ u\in\mathcal{T}.$$

A planned lead time v_{jq} has to elapse between jobs *on the same part*





$$\sum_{k\in\tilde{\mathcal{K}}}\left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}^{\mathrm{pm}}_{jq}} x_{qk\nu}\right) \geq 0, \qquad (j,q)\in\mathcal{Q},\ u=0,\dots,T-\tilde{v}^{\mathrm{pm}}_{jq},$$

$$x_{jku}=0, \qquad (j,q)\in\mathcal{Q},\ k\in\tilde{\mathcal{K}},\ u=T-\tilde{v}^{\mathrm{pm}}_{jq},\dots,T,$$

Time–indexed formulation with nail variables

$$\text{Minimize} \quad \sum_{j \in \mathcal{J}} (A s_j + B h_j),$$

Objective: Minimize the weighted sum of completion times and tardiness.

$$\text{subject to} \quad \sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} x_{jku} = 1,$$

One job is scheduled only once

$$\sum_{u \in \mathcal{T}} x_{jku} \le \lambda_{jk},$$

Each job can only be assigned to an allowed resource *k*

$$\sum_{j \in \mathcal{J}} \sum_{\nu = (u - \tilde{p}_j + 1)_+}^{u} x_{jk\nu} \le 1,$$

Only one job at a time can be processed on resource *k*

$$\sum_{k \in \tilde{\mathcal{K}}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u + \tilde{v}_{jq}^{\text{pm}}} x_{qk\nu} \right) \ge 0,$$

Planned lead time between jobs on same part

$$x_{jku} = 0,$$

The starting time of job j

$$\sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} u x_{jku} + \tilde{p}_j^{\text{pm}} = s_j,$$

Definition of completion time

$$h_j \ge \max\{s_j - \tilde{d}_j; 0\},$$

Definition of tardiness

$$x_{jku} = 0, \qquad j \in \mathcal{J},\ k \in \tilde{\mathcal{K}},\ u = 0, \ldots, \max\{\tilde{r}_j^{\text{m}}; \tilde{a}_k\},$$

$$x_{jku} \in \{0,1\},$$

Job *j* cannot start in resource *k* before the job's release date or before *k* is available

The engineer's model
(with continuous time variables)

$$z_{jk} = \begin{cases} 1, & \text{if job } j \text{ is scheduled on } k \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{jqk} = \begin{cases} 1, & \text{if job } j \text{ is processed before job } q \text{ on } k \\ 0, & \text{otherwise.} \end{cases}$$

Binaries: $|\mathcal{J}||\mathcal{K}| + |\mathcal{J}||\mathcal{J}||\mathcal{K}|$ Test case No 3 (20 jobs): 2100

Time variables

$$t_j = \text{ starting time}$$

$$s_j = t_j + p_j + p_j^{\mathrm{pm}} \text{ completion time of job } j$$

$$h_j = \max\{0, s_j - d_j\} \text{ tardiness of job } j$$

The engineer's model

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j \in J} (As_j + Bh_j), & \\
\text{subject to} \quad & \sum_{k \in \tilde{\mathcal{K}}} z_{jk} = 1, & j \in \mathcal{J}, \\
& z_{jk} \le \lambda_{jk}, & j \in \mathcal{J},\ k \in \tilde{\mathcal{K}}, \\
& y_{jqk} + y_{qjk} \le z_{jk}, & j, q \in \mathcal{J},\ j \neq q,\ k \in \tilde{\mathcal{K}}, \\
& y_{jqk} + y_{qjk} + 1 \ge z_{jk} + z_{qk}, & j, q \in \mathcal{J},\ j \neq q,\ k \in \tilde{\mathcal{K}}, \\
& t_j + p_j - t_q \le M(1 - y_{jqk}), & j, q \in \mathcal{J},\ j \neq q,\ k \in \tilde{\mathcal{K}}, \\
& t_j \ge r_j^{\mathrm{m}}, & j \in \mathcal{J}, \\
& t_j \ge a_k z_{jk}, & j \in \mathcal{J}, \\
& t_q \ge s_j + v_{jq}^{\mathrm{m}}, & (j, q) \in \mathcal{Q}, \\
& s_j - t_j = p_j + p_j^{\mathrm{pm}}, & j \in \mathcal{J}, \\
& h_j \ge \max\{s_j - d_j; 0\}, & j \in \mathcal{J}.
\end{aligned}
$$

weights (A=B=1)

Big number

The engineer's model

$$\text{Minimize} \quad \sum_{j\in J}(As_j + Bh_j),$$

weights (A=B=1)

Objective: Minimize the weighted sum of completion times and tardiness.

$$\text{subject to} \quad \sum_{k\in\tilde{\mathcal{K}}} z_{jk} = 1,$$

One job is scheduled only once

$$z_{jk} \le \lambda_{jk},$$

Each job can only be assigned to an allowed resource *k*

$$y_{jqk} + y_{qjk} \le z_{jk},$$
$$y_{jqk} + y_{qjk} + 1 \ge z_{jk} + z_{qk},$$
$$t_j + p_j - t_q \le M(1 - y_{jqk})$$

Only one job at a time can be processed on resource *k*

Big number

$$t_j \ge r_j^{\mathrm{m}},$$
$$t_j \ge a_k z_{jk},$$

Job *j* cannot start in resource *k* before the job's release date or before *k* is available

$$t_q \ge s_j + v_{jq}^{\mathrm{m}},$$

Planned lead time between jobs on same part

$$s_j - t_j = p_j + p_j^{\mathrm{pm}},$$

Definition of completion time

$$h_j \ge \max\{s_j - d_j; 0\},$$

Definition of tardiness

Comparison between the precedence constraints for the set $\mathcal{Q}$

$$\sum_{k \in \tilde{\mathcal{K}}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}^{\mathtt{pm}}_{jq}} x_{qk\nu} \right) \geq 0,$$
$$x_{jku} = 0,$$

$$t_q \geq s_j + v^{\mathtt{m}}_{jq},$$

Constraints:	$\lvert\mathcal{Q}\rvert \lvert\mathcal{T}\rvert$	$\lvert\mathcal{Q}\rvert$
Test case No 3:	372	4

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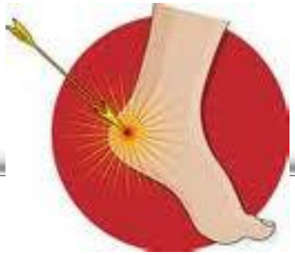
Comparison between the constraints for only one job at a time

$$\sum_{j \in \mathcal{J}} \sum_{\nu=(u-\tilde{p}_j+1)_+}^{u} x_{jk\nu} \leq 1,$$

$$y_{jqk} + y_{qjk} \leq z_{jk},$$
$$y_{jqk} + y_{qjk} + 1 \geq z_{jk} + z_{qk},$$
$$t_j + p_j - t_q \leq M(1 - y_{jqk}),$$

Constraints:	$\lvert\mathcal{K}\rvert\lvert\mathcal{T}\rvert$	$3\lvert\mathcal{J}\rvert\lvert\mathcal{J}\rvert\lvert\mathcal{K}\rvert$
Test case No 3:	465	6000



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Symmetric constraints versus ordering constraints
(the engineer's model)

These two constraints regulates the ordering of the jobs in resource k

The ordering constraints can be replaced by constraints symmetric to ”the big *M*-constraints”:

$$y_{jqk} + y_{qjk} \leq z_{jk},$$
$$y_{jqk} + y_{qjk} + 1 \geq z_{jk} + z_{qk},$$
$$t_j + p_j - t_q \leq M(1 - y_{jqk}),$$

$$t_q + p_q - t_j \leq M(y_{jqk})$$

- The symmetric constraints are common in text books since the ordering constraints are special for problems with multiple machines.
- Computational tests indicate that the model with ordering constraints has **shorter computation times** than a model using the symmetric big *M*-constraints.

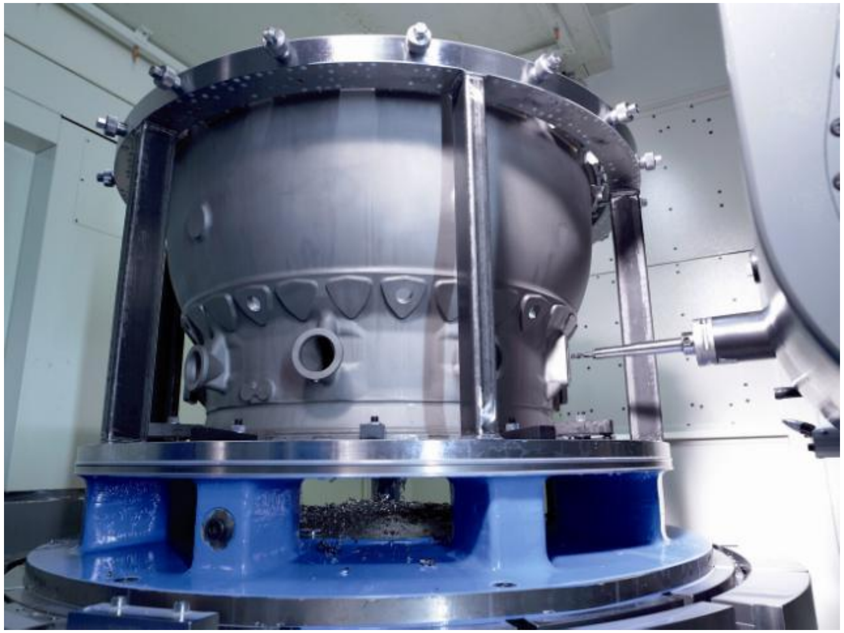
What is the size of a realistic scenario? **45 jobs**

Number of storage locations without fixture: 30

Number of parts arriving during the coming shift: 15

Real production scenarios

- 6 real scenarios – based on real production data extracted from the Volvo Aero ERP-system during the autumn of 2010
- The jobs were ordered according to increasing release dates
- From each scenario test instances were created with 5,10,15,…, 70 of the first jobs in the sorted list of jobs (i.e. the queue of jobs)



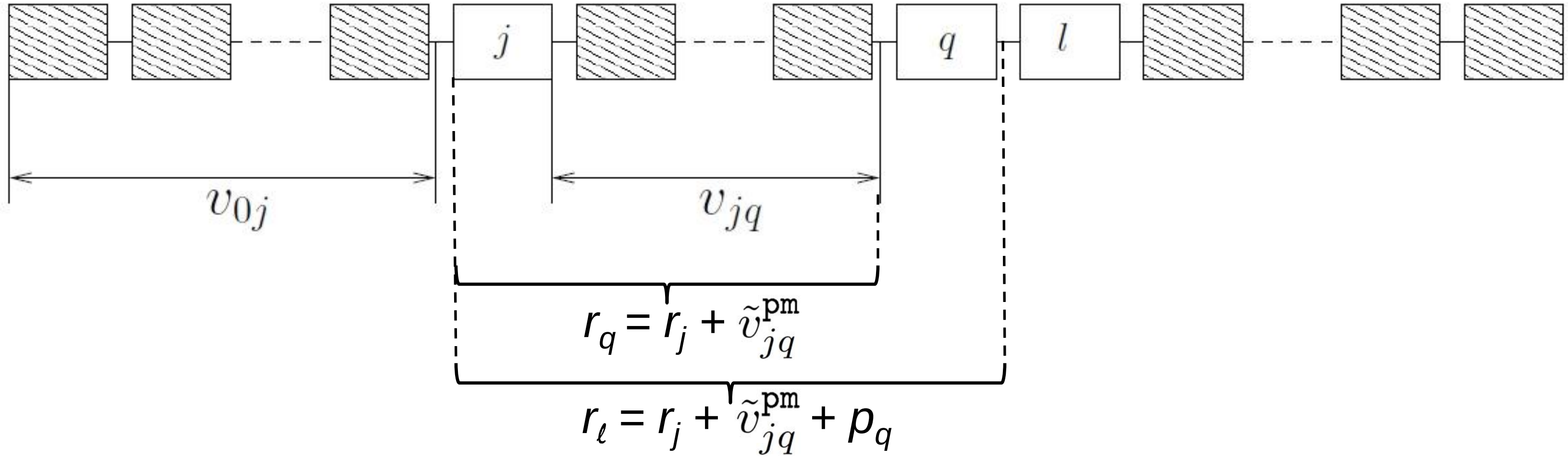
Realistic release dates

The release date from the ERP system may be negative (i.e. in the past). Therefore a realistic estimate on the part arrival time at the multitask cell is calculated using the knowledge of the part's actual position at time t_0. The release date is then calculated by the formula

Standard queue time

$$r_j = \max\{\textit{realistic estimate; ERP release date} - 0.8 \upsilon_j\}$$

If order checked-in: $r_j = 0$



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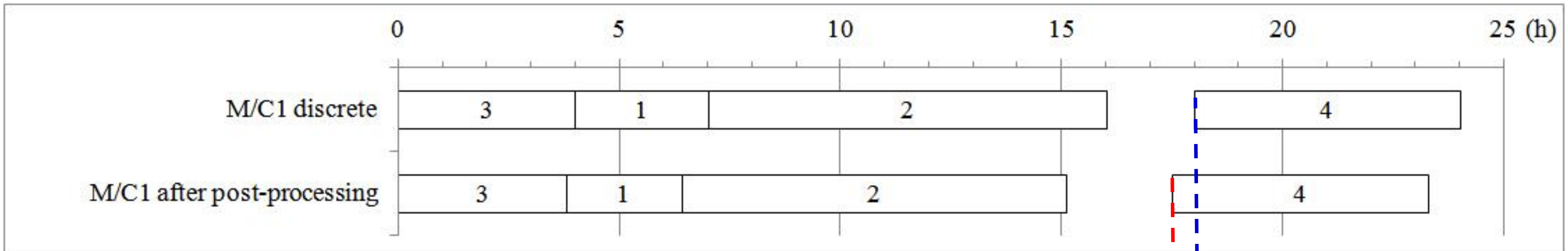
Postprocessing with real ”undiscrete” data



Real release date = 17.5 h

Time-indexed model with ℓ = 1 h: 18 h

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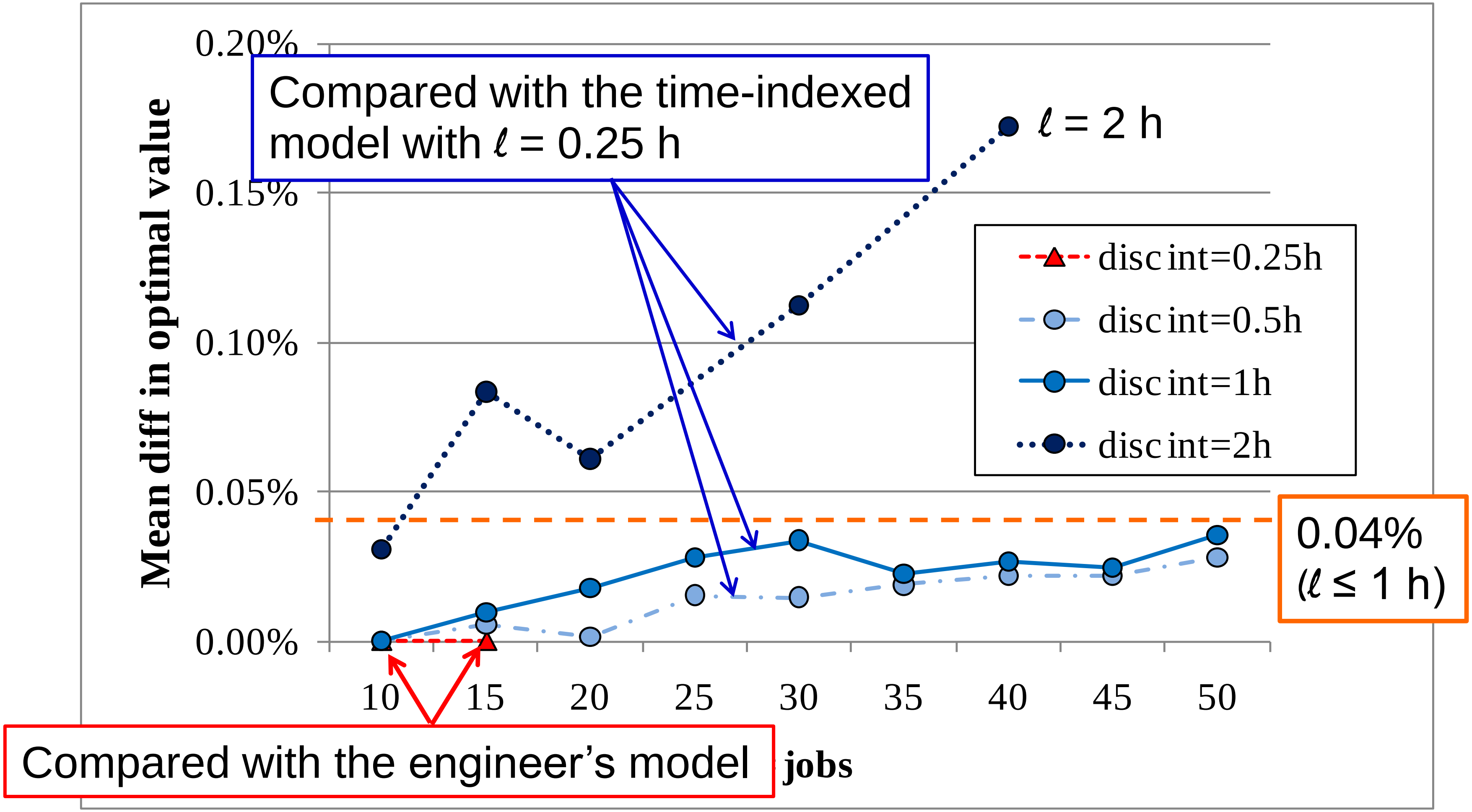
Mean differences between optimal objective values after postprocessing of data



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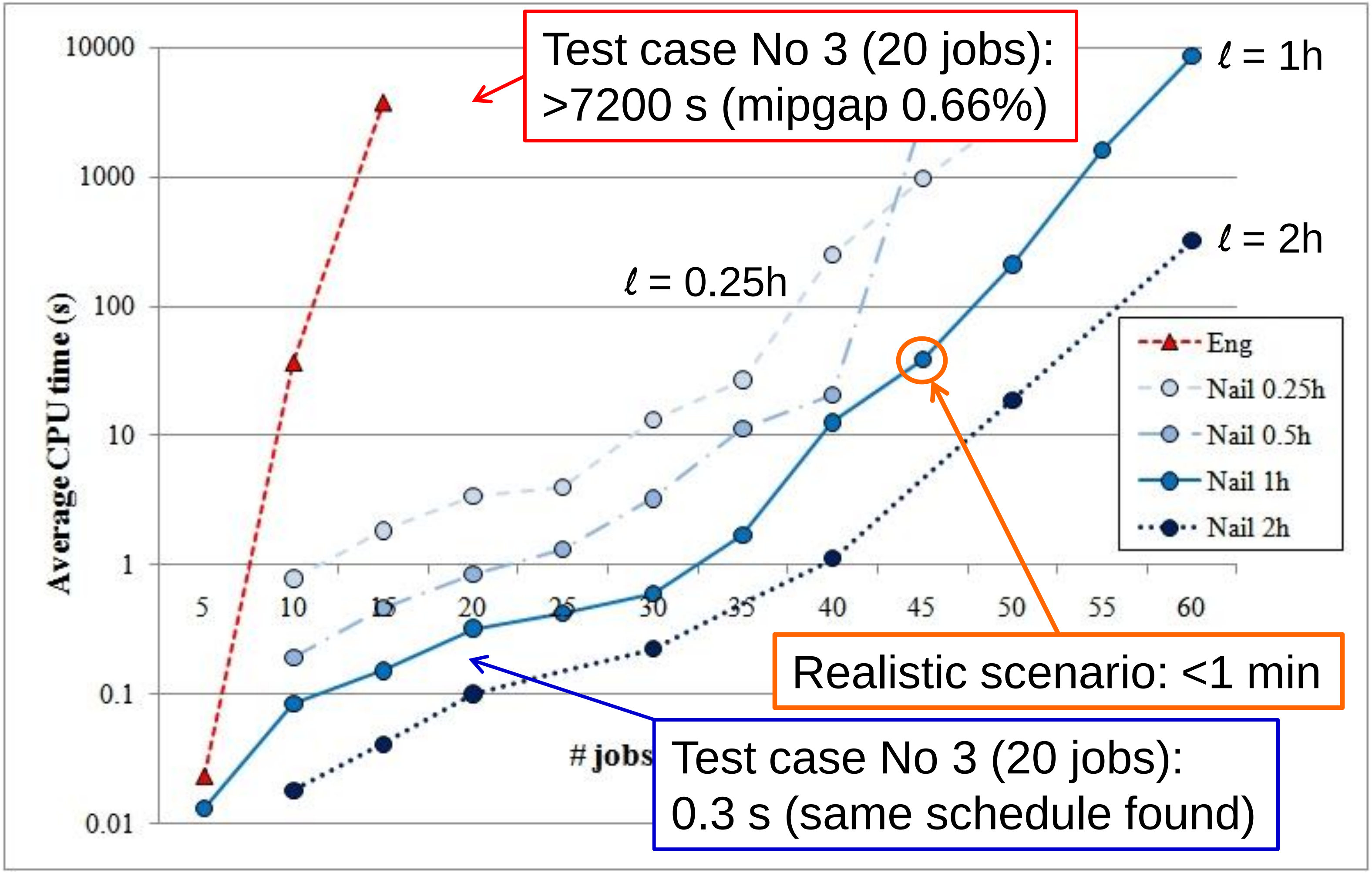
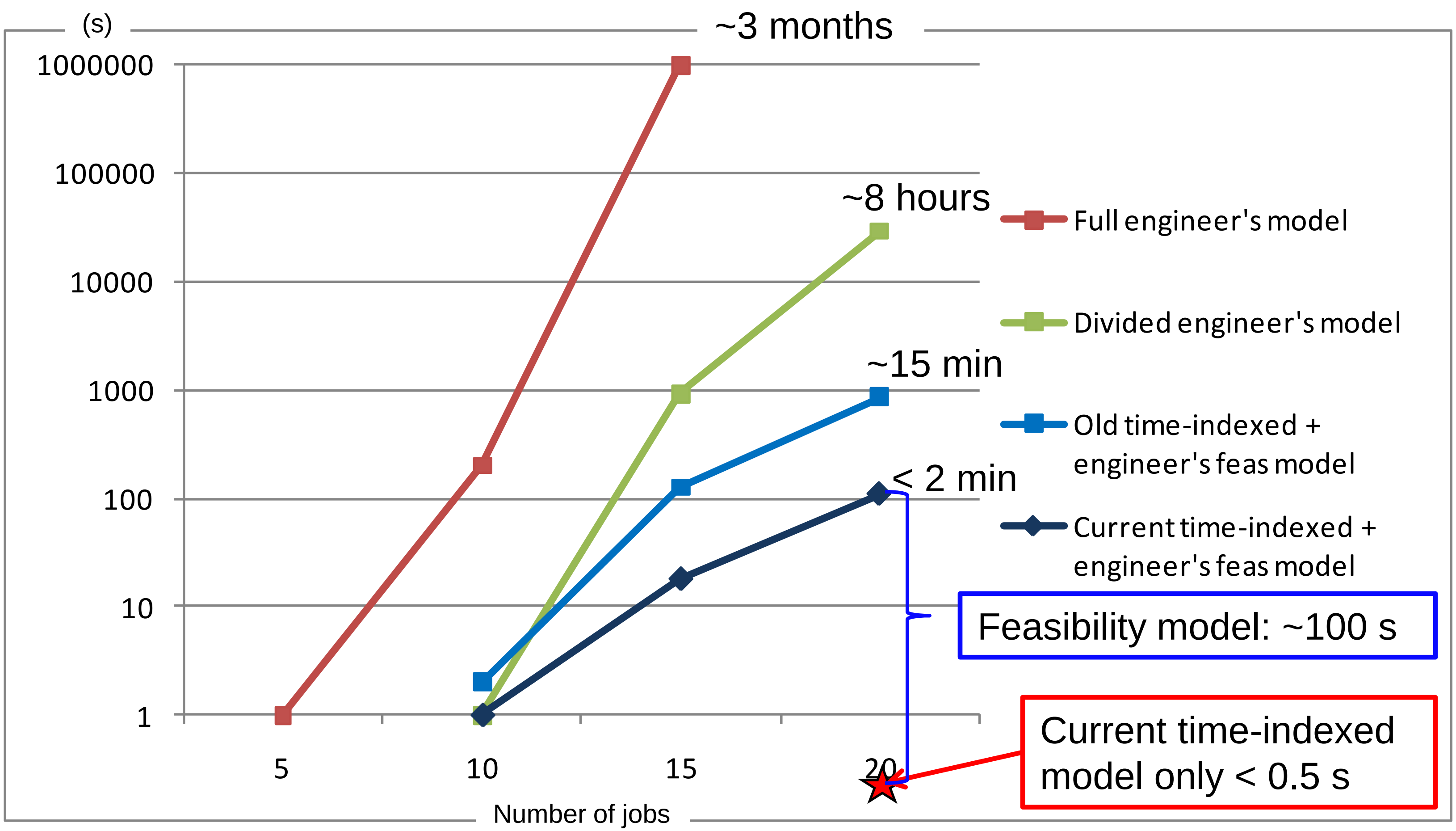
Mean computation times



Evolution of computation times

Comparison between different models' CPU times (seconds)



The feasibility model

different weights for different setup stations

$$
\begin{aligned}
&\text{Minimize} && \sum_{j\in\mathcal{J}}\Big(As_j - \varepsilon t_{1j} + Bh_j + \sum_{i\in\mathcal{N}_j}\sum_{k\in\mathcal{K}}\omega_k z_{ijk}\Big), && \\
&\text{subject to} && \sum_{k\in\mathcal{K}} z_{ijk} = 1, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\\
&&& z_{ijk} \le \lambda_{ijk}, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\ k\in\mathcal{K},\\
&&& y_{ijpqk} + y_{pqijk} \le z_{ijk}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\\
&&& && (i,j)\neq(p,q),\ k\in\mathcal{K},\\
&&& y_{ijpqk} + y_{pqijk} + 1 \ge z_{ijk} + z_{pqk}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\\
&&& && (i,j)\neq(p,q),\ k\in\mathcal{K},\\
&&& t_{ij} + p_{ij} - M(1 - y_{ijpqk}) \le t_{pq}, && i\in\mathcal{N}_j,\ p\in\mathcal{N}_q,\ j,q\in\mathcal{J},\\
&&& && (i,j)\neq(p,q),\ k\in\mathcal{K},\\
&&& t_{ij} + p_{ij} + w \le t_{i+1,j}, && i\in\mathcal{N}_j\setminus\{n_j\},\ j\in\mathcal{J},\\
&&& t_{1j} \ge r_j, && j\in\mathcal{J},\\
&&& t_{ij} \ge a_k z_{ijk}, && j\in\mathcal{J},\ k\in\mathcal{K},\\
&&& t_{1q} \ge s_j + v_{jq}, && (j,q)\in\mathcal{Q},\\
&&& s_j - t_{n_j j} = p_{n_j j}, && j\in\mathcal{J},\\
&&& h_j \ge \max\{s_j - d_j; 0\}, && j\in\mathcal{J},\\
&&& t_{ij} \ge 0, && i\in\mathcal{N}_j,\ j\in\mathcal{J},\\
&&& y_{2j2qk} = y^{\mathrm{m}}_{jqk}, && j,q\in\mathcal{J},\ k\in\tilde{\mathcal{K}},\\
&&& z_{2jk} = z^{\mathrm{m}}_{jk}, && j\in\mathcal{J},\ k\in\tilde{\mathcal{K}}.
\end{aligned}
$$

Fixed to solution from machining problem

where y^m_{jqk} and z^m_{jk} are the solutions obtained from the machining problem.

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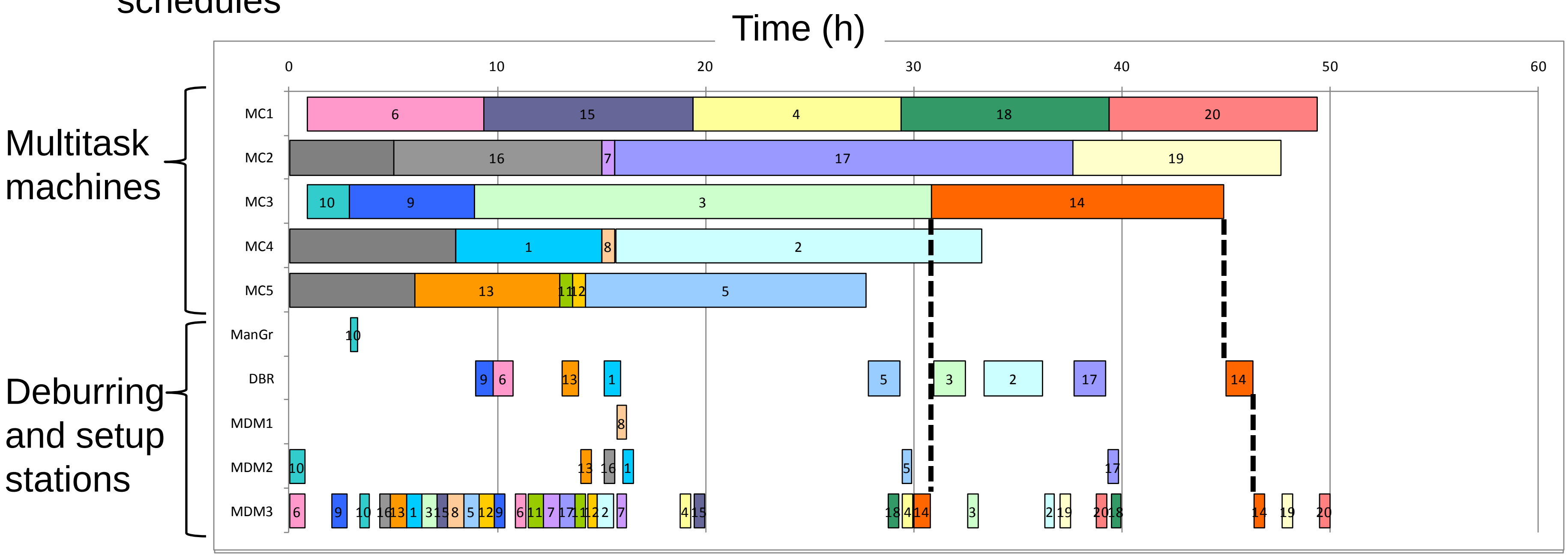
Current detail planning of the multitask cell

Manual planning based on

- Earliest Due Date (EDD) priority list
- Other priorities based on the current logistical situation
- The FIFO priority rule (First In First Out) is used in other parts of the factory
- SPT (shortest processing time) is a priority rule known to produce good schedules



The deburring and set-up stations are scheduled by the use of the feasibility model.

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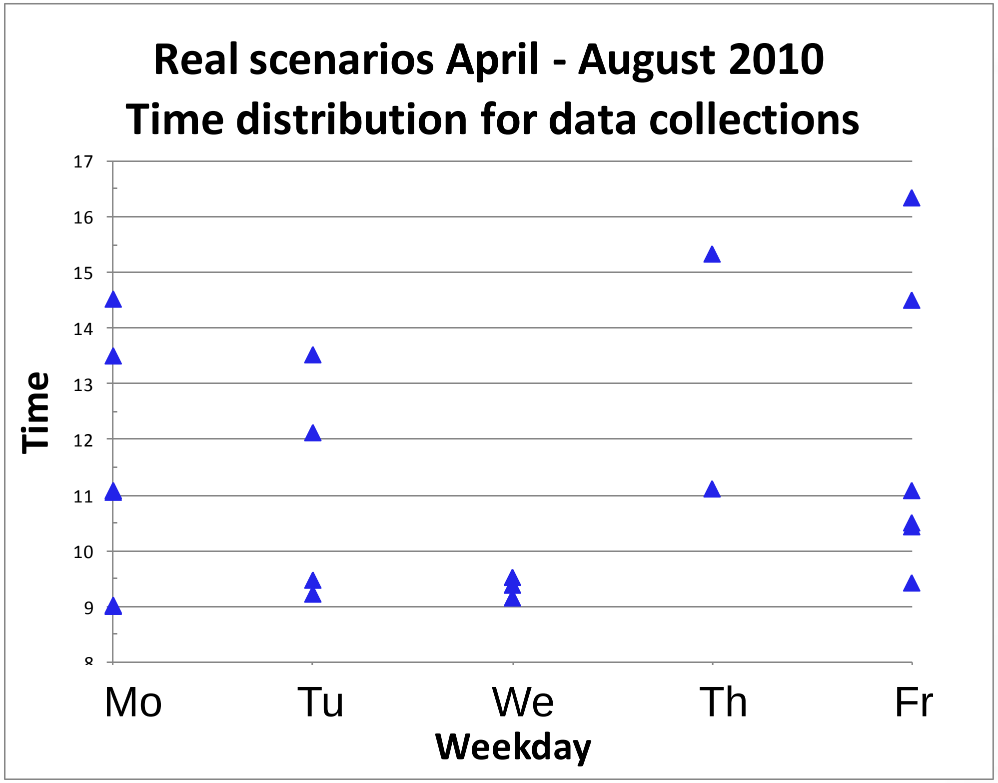
EDD, FIFO and SPT versus mathematical optimization

21 scenarios with real production data
Collected during April – August 2010



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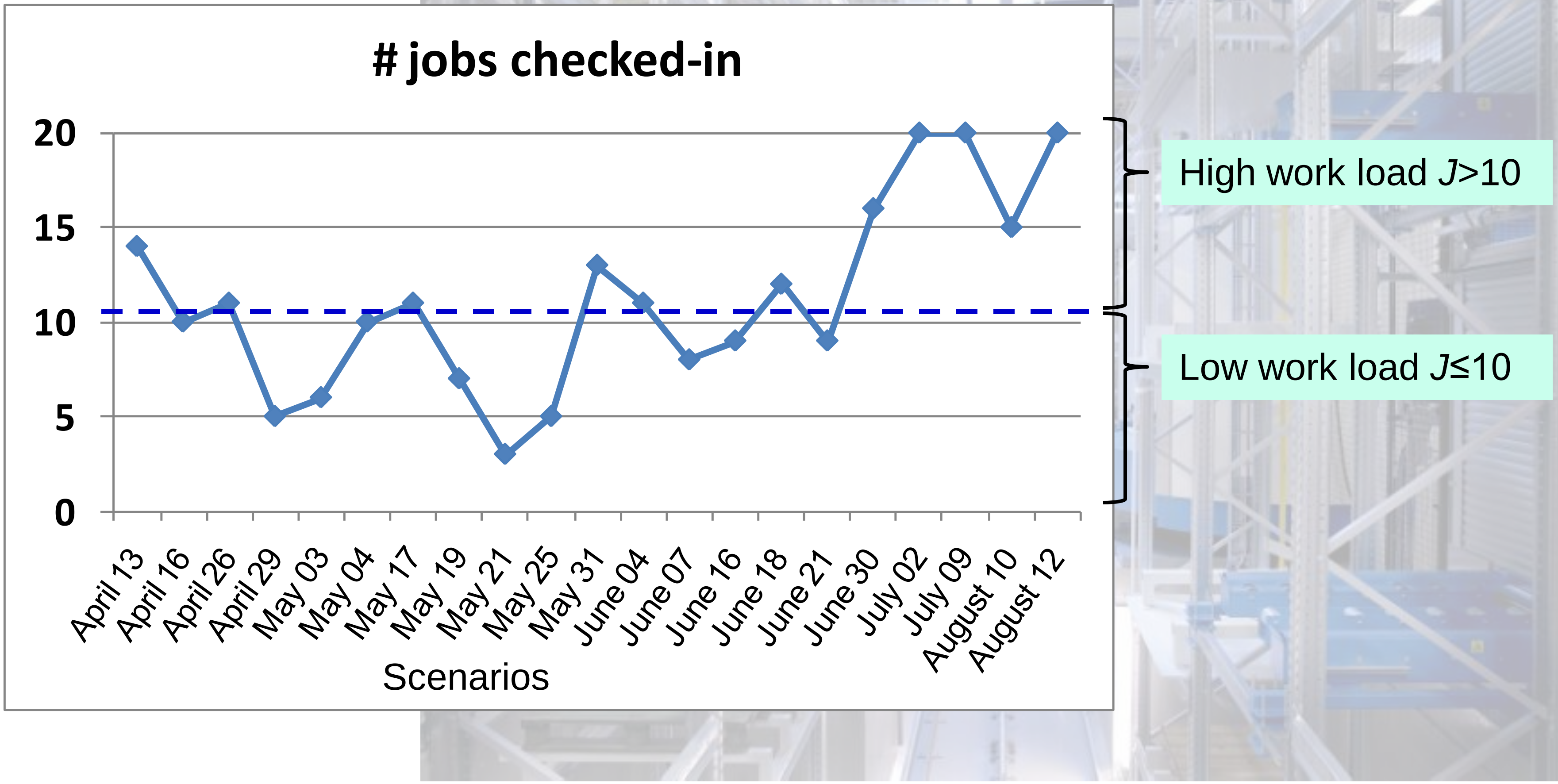
Work load variation

The variation in number of jobs checked-in indicate how the work load has varied during the period.



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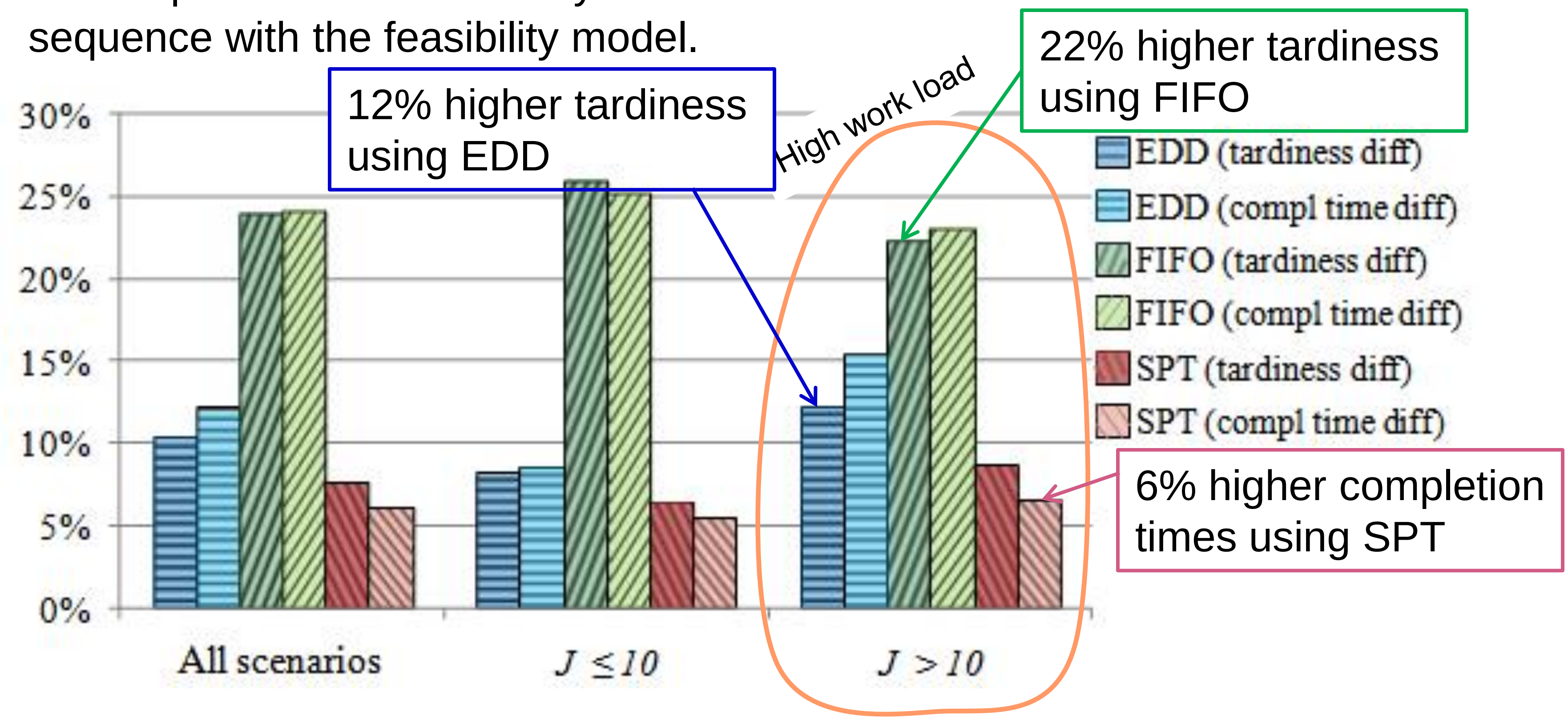
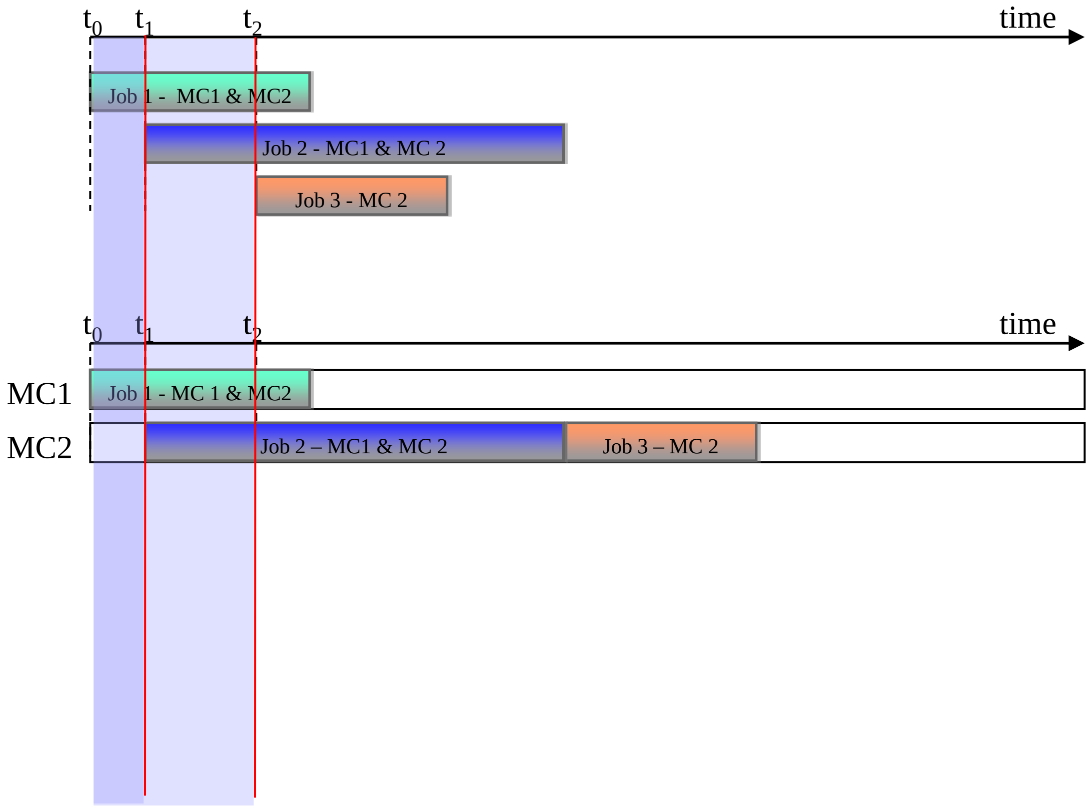
Mean differences between completion times and tardiness results

The schedules resulting from the use of the priority rules are compared to the optimal values found by the time-indexed model with ℓ=1h in sequence with the feasibility model.



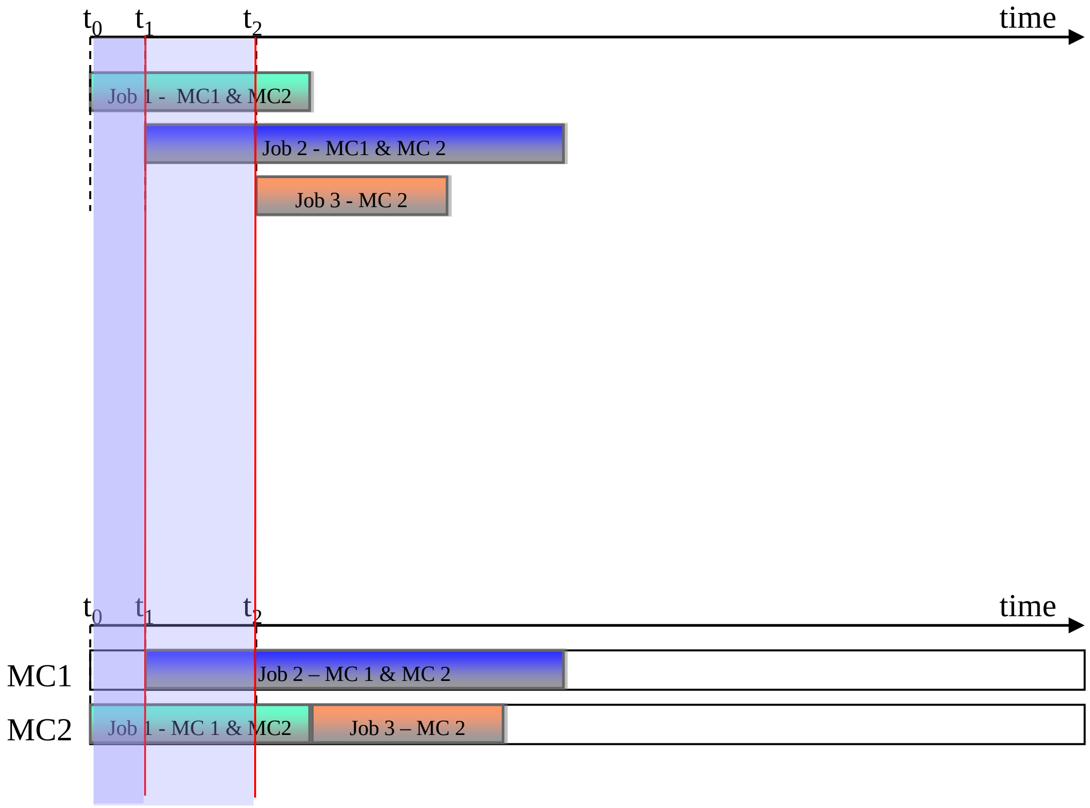
Shortsighted scheduling

No knowledge about which jobs are on the way to the multitask cell (or further down in the priority list)



Looking into the future…

The optimization model takes all jobs in the queue into account



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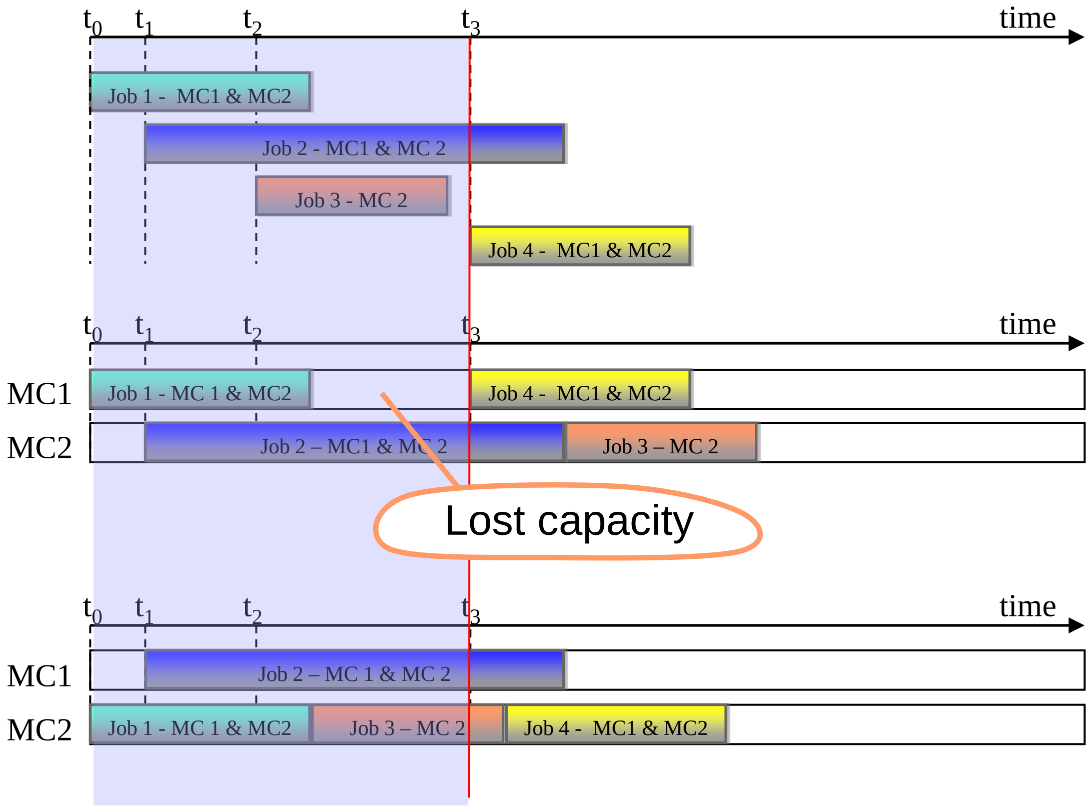
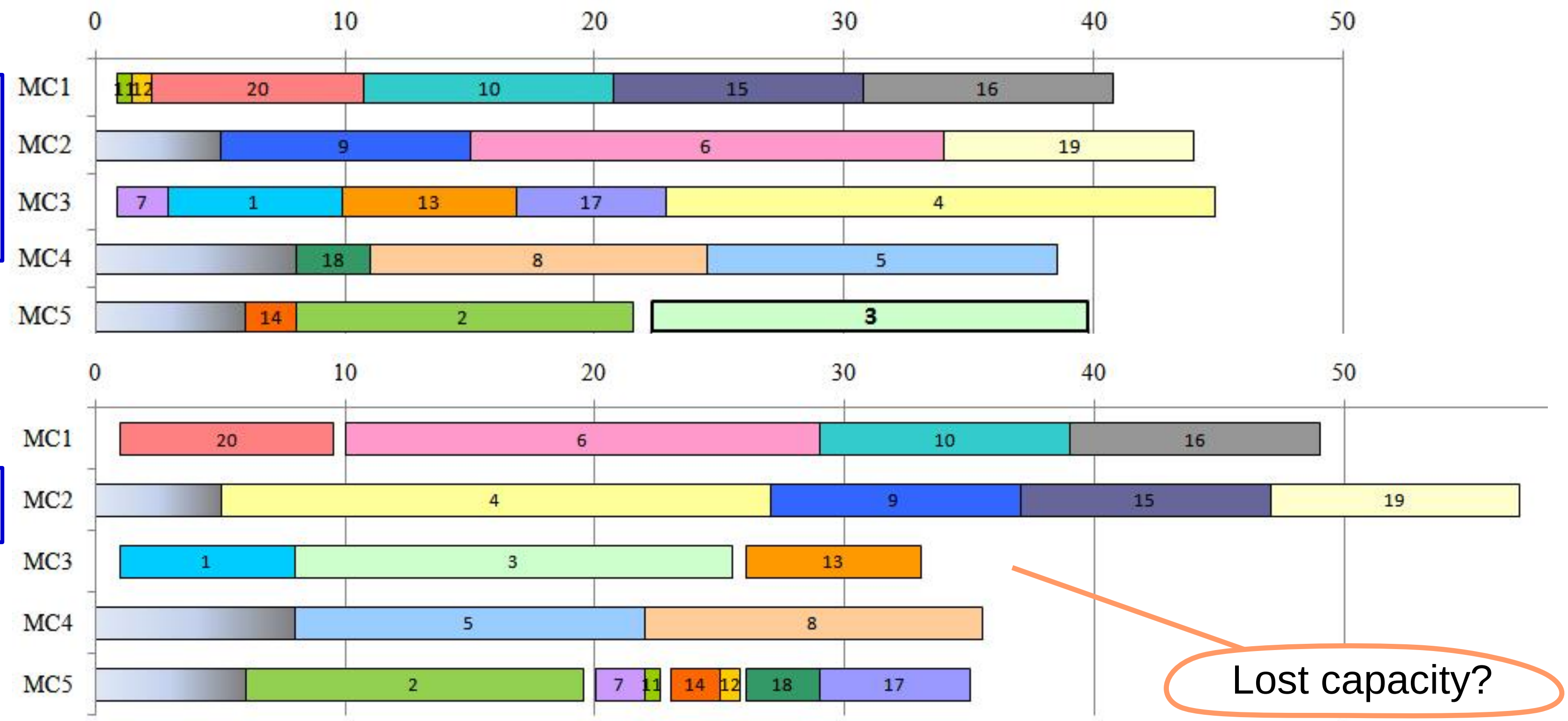
Looking into the future…



An optimal schedule versus a schedule created using the EDD rule

Time-indexed model

EDD



9% higher tardiness using EDD and 25% higher completion times!

Coping with reality

As soon as the production schedule is optimized – something changes!

Expected events

- New parts in the queue
- Variances in the planned lead time

Unexpected events

- Machine breakdown
- Operator sick
- Part with non-conformance leaves queue
- etc.

Frequency:	when necessary
CPU time	< 5 min

Future research

- Compare results to more sophisticated scheduling algorithms
- Make a pilot test of using the schedules in the multitask cell

More tests

- Constraint programming
- Lagrangian relaxation to get better lower bounds
- Column generation
- Find stronger formulations of the problem
- Robust optimization

More theory

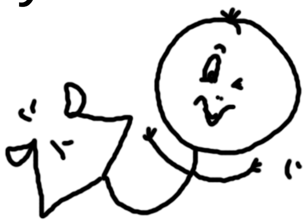
- More realistic model: fixtures, personnel etc.
- Find better objective functions
- Develop a time-indexed formulation for the feasibility problem

Better model

- …

Questions and comments?

Thank you



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