F1 Linjär algebra och geometri, facit till gamla tentor

1993–01–08

1) $l' : \frac{1}{7}(-34,-18,17)+t(24,9,-5)$ **2)** $\lambda \neq 3$: ingen lösning. $\lambda = 3, \mu = 2 : \boldsymbol{x} = (6t/5-1/5, 2t/5+3/5, t, 0)$. $\lambda = 3, \mu \neq 2 : \boldsymbol{x} = (-1/5, 3/5, 0, 0)$. **3)** $z_1 = \frac{1}{2}$, $z_2 = \frac{1}{3}$, $z_{3,4} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$. **4)** $A^{2n} = 2^{2n}E$, $A^{2n+1} = 2^{2n}A$, $n = 0, 1, 2, \ldots$. **5)** Infallande stråle: $(-2,-4,1)$, reflekterad stråle: $(5,10,-8)$.

1993–09–09

1) $5x - 18y + 17z + 8 = 0$ **2)** $\lambda \neq 0, \lambda \neq 1 : \boldsymbol{x} = (0, 2/\lambda, 3/\lambda, (\lambda-5)/\lambda)$. $\lambda = 0$: ingen lösning. $\lambda = 1 : \boldsymbol{x} = (p, 2, 3, -4-p)$ **3)** $z_{1,2} = \pm 3i$, $z_3 = 1$, $z_4 = -2$, $z_5 = -6$. **4)**

$$A^{-1} = \frac{1}{n-1}\begin{pmatrix} 2-n & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2-n & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2-n & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 2-n \end{pmatrix}$$

5) a) — b) $X = -\sum_{m=1}^{k} A^m$

1994–09–08

1) $2x - y - z + 2 = 0$ **2)** $\lambda = -2, \mu \neq 0$: inga lösningar. $\lambda = -2, \mu = 0 : \boldsymbol{x} = (19+7s, -5-2s, -12-5s, s)$ $\lambda \neq -2 : \boldsymbol{x} = (19+7\mu/(\lambda+2), -5-2\mu/(\lambda+2), -12-5\mu/(\lambda+2), \mu/(\lambda+2))$. **3)** a) $\sqrt[10]{2}\,\{\cos[(\pi/6+2k\pi)/10] + i\sin[(\pi/6+2k\pi)/10]\}$, $k = 0, 1, \ldots, 9$. b) $z = ki$, $|k| \leq 1$. **5)**

$$X = \begin{pmatrix} 1 & -1 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 2 \end{pmatrix}$$

6) $(x_1^2 - 1)\cdots(x_n^2 - 1) = (1 + a_{n-1} + \cdots + a_1 + a_0)[(-1)^{n-1}a_{n-1} + \cdots - a_1 + a_0]$

1994–11–04

1) $6x+10y-10z+5 = 0$ **2)** $\lambda = 0$: ingen lösning. $\lambda \neq 0, \lambda \neq 1 : \boldsymbol{x} = (-2-6/\lambda+3r, 5-2r, r, 1, 2/\lambda)$ $\lambda = 1 : \boldsymbol{x} = (3s-4t-4/\lambda, 3-2s+2t, s, t, 2)$. **3)** $z_1 = 3i$, $z_2 = 2-i$, $z_3 = 1+i$. **4)** a) $z \neq \frac{1}{2} \Rightarrow D(z) = [1-(2z)^{n+1}]/(1-2z)$. $z = \frac{1}{2} \Rightarrow D(z) = n+1$, b) $D(z) = 0 \Leftrightarrow z = \frac{1}{2}\exp[i2k\pi/(n+1)]$, $k = 1, 2, \ldots, n$

1994–01–07

1) $5x + 7y + 9z - 44 = 0$ **2)** $\lambda = \mu$: ingen lösning. $\lambda \neq \mu, \mu \neq -1 : \boldsymbol{x} = (4-5/(\lambda-\mu), 1, 1/(\lambda-\mu), 1/(\lambda-\mu), 1)$. $\lambda \neq \mu, \mu = -1 : \boldsymbol{x} = (4-5/(\lambda-\mu), t, 1/(\lambda-\mu), 1/(\lambda-\mu), t)$. **3)** $z_{1,2} = -1 \pm 2i$, $z_{3,4} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. **4)** $z_k = 3i(\exp[i(3\pi/2+2k\pi)/n]+1)/(\exp[i(3\pi/2+2k\pi)/n]-1)$.

1995-08-29

1) $l : (0,1,-1) + t(3,-13,29)$. **2)** $\lambda \neq -1$: ingen lösning. $\lambda = -1 : \boldsymbol{x} = (0, 3t-1/2, 2t, 0)$. **3)** $z_{1,2} = \pm 2i$, z_3=2, $z_{4,5} = \frac{1}{2} \pm \frac{\sqrt{11}}{2}i$. **5)** Ja. **6)** b) Nej.

1999-10-23

1) $P'\,(\frac{1}{5},-\frac{3}{5},1)$. **2)** $\mu \neq 6$: entydig lösning $\forall\lambda$, $\boldsymbol{x} = (\frac{4\lambda-11}{\mu-6}, 2-2\lambda-2\frac{4\lambda-11}{\mu-6}, 4-\frac{4\lambda-11}{\mu-6}, \frac{4\lambda-11}{\mu-6})$. $\mu = 6, \lambda \neq \frac{11}{4}$: ingen lösning. $\mu = 6, \lambda = \frac{11}{4}$: $\boldsymbol{x} = (\frac{25}{4}+t, -\frac{7}{2}-2t, 4-t, t)$. **4)** $z_{1,2} = -1 \pm i$, $z_{3,4} = 1 \pm \sqrt{2}$. **5)** $\lambda = -\frac{2}{n(n+1)}$.

2000-01-14

1) $2x - y + z + 12 = 0; S\,(-8, 4, -4)$. **2)** $\lambda \neq -2; 2; 4; 6$: entydig lösning $\boldsymbol{x} = (0,0,0,0)$. $\lambda = -2$: $\boldsymbol{x} = (t,t,t,0)$. $\lambda = 2 : \boldsymbol{x} = (s,0,0,s)$. $\lambda = 4 : \boldsymbol{x} = (0,p,0,0)$. $\lambda = 6 : \boldsymbol{x} = (0,0,q,q)$. **3)** $z_1 = 2$, $z_2 = 1-2i$, $z_3 = -2+i$. **4)** $\lambda = 15$. **5)** $n(-1)^{n-1}$.

2000-08-16

1) $l : (1,0,7) + t(68,70,-67); S\,(\frac{122}{21}, \frac{257}{42}, \frac{71}{84})$. **2)** $\lambda = 0 : \boldsymbol{x} = (-\frac{17}{36}-t, -\frac{17}{36}-t, \frac{7}{9}-t, 9t)$. $\lambda = -2 : \boldsymbol{x} = (\frac{19}{6}-3s, -\frac{5}{2}+s, s, s)$. **3)** $z_1 = 5i$, $z_{2,3} = -\frac{1}{2}$, $z_{4,5} = \frac{1 \pm i\sqrt{5}}{2}$. **4)** $n > 2 : 0; n = 1 : 1 + x_1y_1; n = 2 : (x_2-x_1)(y_2-y_1)$. **6)** Punkterna i I kvadranten, som ligger på linjen $bx + ay - 2S = 0$, där $|AB| = a, |CD| = b$.