TMA682, Extra övningar i Laplacetransform

Use Laplace transforms to solve the following initial-value problems:

1. $y'' - y = 1, \qquad y(0) = 0, \quad y'(0) = 1.$
   answer: $y = e^t - 1.$
2. $y'' - 3y' + 2y = 0, \qquad y(0) = 3, \quad y'(0) = 4.$
   answer: $y = e^{2t} + 2e^t.$
3. $4y'' + y = -2, \qquad y(0) = 0, \quad y'(0) = 1/2.$
   answer: $y = -2 + 2\cos(t/2) + \sin(t/2).$
4. $y'' + 2y' + y = e^t, \qquad y(0) = 0, \quad y'(0) = 0.$
   answer: $y = \frac{1}{4}e^t - \frac{1}{4}e^{-t} - \frac{1}{2}te^{-t}.$
5. $y'' + 2y' + 3y = 3t, \qquad y(0) = 0, \quad y'(0) = 1.$
   answer: $y = \frac{2}{3}e^{-t}\cos\sqrt{2}t + \frac{\sqrt{2}}{3}e^{-t}\sin\sqrt{2}t + t - \frac{2}{3}.$

Find the inverse Laplace transform of the following functions:

6. $\frac{1}{s(s+2)^2}$ answer: $\frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}te^{-2t}.$
7. $\frac{1}{s^2+4s+29}$ answer: $\frac{1}{5}e^{-2t}\sin 5t.$
8. $\frac{2s}{(s^2+1)^2}$ answer: $t\sin t.$
9. $\frac{3s^2}{(s^2+1)^2}$ answer: $\frac{3}{2}\sin t + \frac{3}{2}t\cos t.$
10. $\ln\frac{s+3}{s+2}$ answer: $\frac{1}{t}(e^{-2t} - e^{-3t}).$