

TMA690 study guide

- Make sure that you read your class notes **at least** once so that you know what was covered.
- You have to be able to state the main theorems of the course: Riesz' representation theorem, the Lax-Milgram Lemma, the trace theorem, Poincare's inequality, Friedrich's inequality, the the Maximum principle for elliptic problems, Poisson's integral formula, the maximum principle for parabolic problems, solution formula for the pure initial value problem for the heat equation, solution formula for the pure initial value problem for the wave equation in 1 and 3 D, Lebesgue's dominated convergence theorem.
- You are expected to know the proof of: the Cauchy-Schwarz inequality, Theorem A.2, the Friedrich's inequality (Problem 3.4), the mean value property of harmonic functions using Poisson's formula, Theorem 3.5 (both 2D and 3D), Theorem 8.1, Theorem 8.5, Theorem 8.6, Theorem 11.2.
- You should be able to explain: Inner product space, Hilbert space, normed space, Banach space, bounded linear functional/operator and their norms, bilinear forms (positive definite, bounded, coercive), Lebesgue measure 0, the space L^2 , essential supremum, the weak derivative of L^2 functions and when does the weak derivative belong to L^2 , Sobolev spaces, the space H_0^1 , the space H^{-1} , the method of Greens's function for boundary value problems, weak solution of boundary value problems, the finite element method, distributions, basic operations with distributions (differentiation, multiplication by a smooth function, convolution with a test function), convergence of distributions.
- You should be able to write various boundary value problems in a weak (variational) form, prove existence and uniqueness of the solutions of the variational problem, show that a smooth weak solution is a classical solution, use energy arguments to derive estimates for solutions of elliptic, parabolic and hyperbolic

problems, use the method of descent, use the method of characteristics to solve boundary value problems for first order linear PDEs.

- Make sure that you know how to do the homework exercises.