LARGE AND SPARSE MATRIX PROBLEMS, 2010

HOMEWORK ASSIGNMENT number 4

Well performed this homework assignment gives 1 credit point

To be handed in by March 10 at the latest

Exercise HA4. Consider a symmetric positive definite 8×8-matrix with sparsity structure

$$A = \begin{bmatrix} x & x & & x & x & & x & \\ x & x & x & & x & & & \\ & x & x & & & x & x & \\ x & & & x & & & & x \\ x & x & & & x & & & \\ & & x & & & x & & \\ x & & x & & & & x & \\ & & & x & & & & x \end{bmatrix}$$

a) Determine the amount of fill-in in the Cholesky-factor by using the graph theory.
b) Determine a RCM-ordering and the fill-in in the Cholesky-factor for this ordering.
c) Determine a minimum-degree ordering and the fill-in in the Cholesky factor for this ordering.
d) Compare your results with MATLAB:s *symrcm* and *symamd*. Use *spy* to illustrate the sparsity patterns of the matrices and their Cholesky factors.

COMPUTER EXERCISE number 4

To be handed in by March 10 at the latest

Exercise CE4. We test different orderings for sparsity for our model Poisson problem in 2D and 3D. The orderings to be tested are **reverse Cuthill-McKee** *symrcm*, **approximate minumum degree** *symamd*, **nested dissection** *nested*, and the standard **columnwise** ordering.
For grade 3: Take n as large as the computer can master with reasonable cpu-time. Compare the different orderings for the 2D and 3D model Poisson problems. Exclude nested dissection from the 3D comparisons. Try to find out how the cputime depends on the number of nonzeros in the Cholesky factor. Do you get an idea of the ordering used by backslash?
Additional for grade 4: With respect to cputime as a function of the problem size n for the model 3D Poisson problem, find the break even point between a direct method with a good ordering and the pcg method with a good preconditioning.
Additional for grade 5: Compare two different nested dissection orderings; the one presented in the lecture notes and the variant used by MATLAB. Preferable you should write a function in MATLAB for the first variant in order to be able to compare for large matrices of proper size. At least you should be able to compare the sparsity of the Cholesky factors for a 17 by 17 (including boundary points), possibly numbered by pen-and-paper.