LARGE AND SPARSE MATRIX PROBLEMS, 2012

HOMEWORK ASSIGNMENT number 5

Well performed this homework assignment gives 1 credit point

To be handed in by March 12 at the latest

Exercise HA5. Let A be real, symmetric and let $K_k(A, v)$ be the Krylov space.
a) Show that $K_k(\beta A, \alpha v) = K_k(A, v),\ \beta \in R,\ \alpha \in R.$
b) Show that $K_k(A - \mu I, v) = K_k(A, v),\ \mu \in R.$
c) Show that $K_k(Q^T AQ, Q^T v) = Q^T K_k(A, v)$ if Q is square, orthogonal.
d) Solve Question 7.1 in the text book.

COMPUTER EXERCISE number 5

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Exercise CE5. We test different iterative methods for eigenproblems. As test matrix we use (once again) the Possion matrix in 2D, corresponding to different meshsizes, grids with $n \times n$ points, including boundary points. The size of the corresponding matrix is then $(n-2)^2 \times (n-2)^2$.
For grade 3: For $n = 9$, compute the two largest and two smallest egenvalues by the Lanczos-Rayleigh-Ritz algorithm without reorthogonalization and with a random start. Sometimes you may need to rerun with a different random start. For solving tridiagonal eigenvalueproblems, simply use *eig* in MATLAB. Present the result in a picture like Fig. 7.2 in the text book. Try to compute all eigenvalues by running 49 iterations. Except from multiplicity, do you get them all? Is your observation in agreement with the theory in the text book?
Additional for grade 4: Now, try to compute all eigenvalues with correct multiplicity. Introduce reorthogonalization in your code. Use the test matrix with $n = 9$ and $n = 17$. How many reorthogonalization sweeps are needed, respectively, in the two cases? Do you get the correct multiplicity of extreme eigenvalues, say the six largest and six smallest, after a reasonable number of iterations? Are your observations in agreement with the theory?
Additional for grade 5: Compare three different methods for computing extreme eigenvalues of a large, sparse, symmetric matrix. Use the test matrix with $n = 33$ and compute the five smallest and the five largest eigenvalues. The three methods are: MATLAB's *eigs*, Lanczos algorithm with reorthogonalization discussed above, and orthogonal iteration i.e. Algorithm 4.3 in the text book. In the latter method, use $p = 5$ and a random start matrix Z_0. Choose a breaking tolerance such that the multiple eigenvalues are computed correctly. Compare the computing time for the different methods. Is your result for orthogonal iteration in agreement with the theory?