Department of
Mathematics
Göteborg

EXAMINATION FOR
LARGE AND SPARSE MATRIX PROBLEMS, TMA891/MMA610
2014-03-13

DATE: Thursday March 13 TIME: 8.30 - 12.30 PLACE: V

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by April 4 at the latest
	Your marked examination can be received at the student's office
	at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments
	Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
a) Show that property(A) implies existence of a consistent ordering of the matrix. **(2p)**
b) Give an example of a consistent ordering for the 2D Poisson's model 5-point matrix. Explain why this ordering represents property(A). **(1p)**

Question 2.
a) Consider the CG method. Use the fact that
$r_k^T p_j = 0, k \neq j$ i.e search directions and residuals are orthogonal
in order to prove that residuals are orthogonal, i.e. $r_{k+1}^T r_j = 0,\ j = 1, \cdots, k$. **(2p)**
b) Describe a polynomial acceleration technique and show that an approximation problem arises. You may assume that the eigenvalues of the iteration matrix are real. **(3p)**.

Question 3.
a) Describe the idea behind the basic multigrid method - the V-cycle. In particular, present the three different operators involved. **(2p)**
b) Give concrete examples of the three operators in a) for the 2D Poisson's model problem. **(2p)**.
c) Draw a picture of a full multigrid iteration. Indicate where the three operators in a) are used in the method. **(1p)**.

Question 4.

Consider the symmetric positive definite matrix $A = \begin{bmatrix} 4 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 4 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 4 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 4 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 4 \end{bmatrix}$.

a) Give a compressed row storage of A. **(2p)**
b) Indicate the envelope of A and where fill-in will occur during Cholesky factorization of A. **(2p)**
c) Use graph-theory to determine a minimum degree ordering for A and the corresponding fill-in in the Cholesky factor. Give the reordered matrix. **(2p)**

Question 5.
a) The Krylov space used for computing eigenvalues to a matrix A is defined by $Span\{v_i\}_{i=1}^{k}$. Give the vectors $\{v_i\}_{i=1}^{k}$. State a property of A such that the scalar product of any two vectors in the set $\{v_i\}_{i=1}^{k}$ is positive. Prove your statement. **(2p)**
b) Consider the Lanczos Rayleigh Ritz method. In what sense are the Ritz values and Ritz vectors optimal approximations? **(2p)**
c) Explain why selective reorthogonalization is used and briefly indicate how it is performed. **(2p)**