EXERCISE 1: MODELLING

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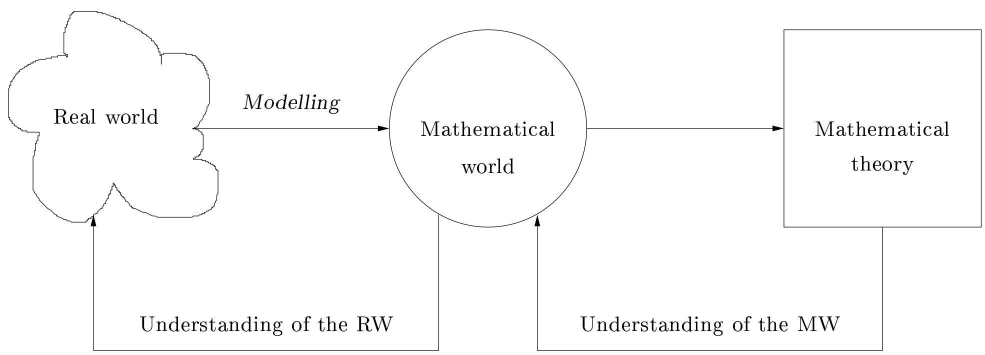
Modelling as a part of the optimization process



To transform a real world problem into an appropriate optimization problem is an art in itself, and unfortunately there is not much theory describing how to do this. The general approach can however be described by two steps:

(1) Prepare a list of all the decision variables of the problem. This list must be complete in the sense that if an optimal solution providing the values of each of the variables is obtained, then the decision maker should be able to translate it into an optimum policy that can be implemented.
(2) Use the variables from step 1 to formulate all the constraints and the objective function of the problem.

EXERCISE 1 (process description). A process is used to produce the product P. In order to produce 1 unit of P the process takes 3 units of the raw material R_1 and 2 units of the raw material R_2. Describe the process mathematically. □

EXERCISE 2 (limited supply of raw material and demand). In order to produce the products P_1 and P_2 two processes are used. Process 1 takes 3 units of R_1 and 1 unit of R_2 to produce 1 unit of P_1, and process 2 takes 2 units of R_1 and 4 units of R_2 to produce 1 unit of P_2. The raw materials R_1 and R_2 cost r_1 and r_2 per unit respectively, and the supply is limited to s_1 and s_2 units. The products P_1 and P_2 can be sold for p_1 and p_2 per unit respectively, and the demand is limited to d_1 and d_2 units. Formulate the problem of choosing the best production strategy as an LP problem. □

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EXERCISE 3 (fixed cost constraints). Consider the problem given in Exercise 2 with the modification that before anything can be produced with the processes 1 and 2 the fixed start up costs c_1 and c_2 respectively must be paid. Modify your LP formulation from Exercise 2 to include these fixed costs. Is the new problem a linear program? □

EXERCISE 4 (the transshipment problem). The transshipment problem is a generalization of the transportation problem (see Example 8.1 in Section 8.1 of the course notes). The transshipment problem considers N sources, M demand centers and I *intermediate* nodes (shipments through intermediate nodes are called *transshipments*). Commodity cannot be shipped directly from a source to a demand center, but must first be shipped through an intermediate node.

Now consider the transshipment problem given in Figure 1. Assume that the availability of commodity at source i is s_i for $i = 1, \dots, 4$, and that the requirements at demand center j is d_j for $j = 1, 2, 3$. Further the cost of shipping one unit of commodity from source i to the intermediate node k is given by a_{ik} and the cost of shipping one unit of commodity from intermediate node k to demand center j is given by b_{kj}. Formulate a linear program for minimizing the total shipping cost. □

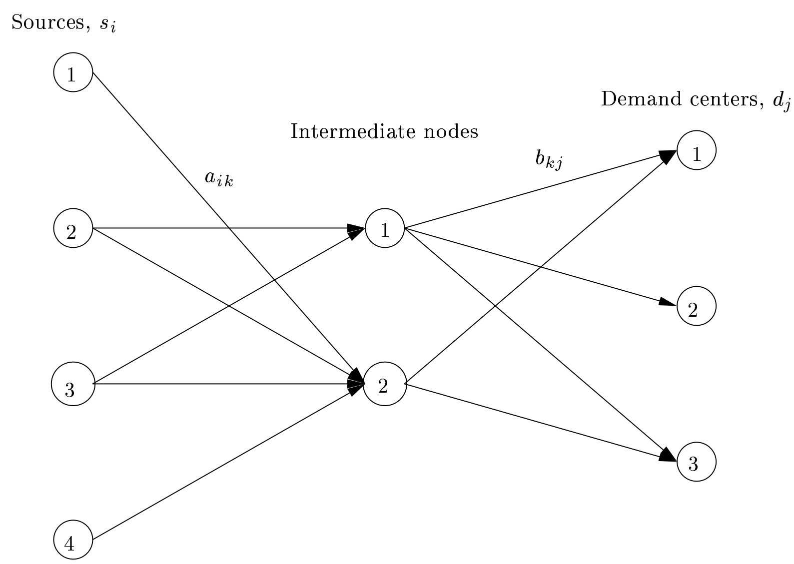

FIGURE 1. Illustration of the transshipment problem.

EXERCISE 5 (logical constraints). Consider the constraints

$$\begin{aligned} \boldsymbol{a}^{\mathrm{T}}\boldsymbol{x} &\leq b, && \text{(I)} \\ \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} &\leq d, && \text{(II)} \\ \boldsymbol{x} &\leq \boldsymbol{1}^n, \\ \boldsymbol{x} &\geq \boldsymbol{0}^n, \end{aligned}$$

where $\boldsymbol{a}, \boldsymbol{c} \in \mathbb{B}^n$ and $b, d \in \mathbb{R}_+$. Now, suppose that only *one* of the constraints (I) and (II) must be fulfilled. Formulate this situation as an integer program. □

EXERCISE 6 (the diet problem [a classic LP]). The following table shows the amount of nutrients in two types of food (grain), the cost of each food, and the daily requirement of each nutrient.

	Food type		
Nutrient	1	2	Minimum daily requirements
Starch	5	7	8
Protein	4	2	15
Vitamins	2	1	3
Cost ($/kg)	0.60	0.35	

Formulate the problem of finding a minimum cost feasible diet as a linear program. □

Stigler's diet problem

In 1945 Stigler, a future Nobel laureate in economics, posed the following problem: For a moderately active man (economist) weighing 154 pounds, how much of each of 77 foods should be eaten on a daily basis so that the man's intake of nine nutrients (including calories) will be at least equal to the recommended dietary allowances suggested by the National Research Council in 1943, with the cost of the diet being minimal?

An optimal solution to Stigler's diet problem

The optimal solution computed with the Simplex method only contains 5 food types!

Food	Annual quantity
Wheat flour	299 lb.
Cabbage	111 lb.
Spinach	23 lb.
Dried navy beans	378 lb.
Beef liver	2.57 lb.
Total daily cost (1998 $)	$1.278

Do you think this is a palatable diet?

Project part I: Modelling

In the industry it is more likely that your model will be used if the managers understand it. One of the purposes of the project is that you should learn how to formulate a complex problem as an easily understandable linear program. Therefore we have the following requirements on your report:

- it shall include a figure illustrating the material flows of the problem;
- the variables must be clearly defined and connected to the figure;
- the objective function and the constraints should be clearly described;
- it must be written with a text formatting tool (e.g., LaTeX, Word, or FrameMaker); and
- it should look professional!

Before starting the modelling work you should read Section 8.1 in the course notes and Example 8.1 in particular.