Exercise 0: Prerequisites

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Exercise 1 Using matrix and vector notation is common in the field of optimization. It is important to be able to perform elementary calculations on such functions.

a) Calculate the gradient $\nabla(x^{\mathrm{T}}Ax)$.

b) Calculate the Hessian $\nabla^2(x^{\mathrm{T}}Ax)$.

Exercise 2 Another important concept in the theory of optimization is the notion of positive definite matrices. Let $A \in \mathbb{R}^{n\times n}$ be a quadratic matrix and λ_i be the eigenvalues of A for $i = 1, \ldots, n$. Show that A is positive (semi) definite if.f. $\lambda_i > 0$ ($\lambda_i \geq 0$) for all $i = 1, \ldots, n$.

Exercise 3 Let $A \in \mathbb{R}^{m\times n}$ be a matrix. Define and/or discuss the following concepts from Linear Algebra.

a) $\mathcal{N}(A)$, the null space of A,

b) $\mathcal{R}(A)$, the range of A,

c) the rank of A,

d) matrices with $m = n$, $m < n$ and $m > n$.

Exercise 4 Let $c \in \mathbb{R}^n$. Define the set $C = \{x \in \mathbb{R}^n : c^{\mathrm{T}}x \leq 0\}$. Let $n = 2$ and $c = (1\ 1)^{\mathrm{T}}$, draw the the set C.