Additional exercises to Exercise 10 - Sensitivity analysis
TMA947 and MMG620 Optimization, first course

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Exercise 1 (easy) Consider the following linear program.

$$\begin{aligned} \text{minimize} \quad & z = -2x_1 + x_2 \\ \text{subject to} \quad & x_1 - 3x_2 \leq \beta, \\ & 0 \leq x_1, \\ & 0 \leq x_2 \leq 2. \end{aligned}$$

Assuming that $\beta \leq 0$, we rewrite the problem on standard form.

$$\begin{aligned} \text{minimize} \quad & z = -2x_1 + x_2, \\ \text{subject to} \quad & -x_1 + 3x_2 - s_1 = -\beta, \\ & x_2 + s_2 = 2, \\ & x_1, \quad x_2, \quad s_1, \quad s_2 \geq 0. \end{aligned}$$

The optimal solution for $\beta = -3$ has x_1 and x_2 as basis variables. What is the marginal change in optimal objective when β is varied from its current value of -3 (i.e. calculate $\frac{\partial z^*}{\partial \beta}$).

Exercise 2 (easy) Consider Exercise H8.4. State for which values of the first component (the one which is 7 now) of the right-hand side vector the optimal basis remains being the optimal one.

Exercise 3 (easy) Consider again Exercise H8.4.

(a) Assume that the cost coefficient of x_1 is modfied to $2 + \varepsilon$. State for which values of ε the current optimal basis remains being the optimal one.

(b) Assume that the cost coefficient of x_2 is modfied to $-1 + \varepsilon$. State for which values of ε the current optimal basis remains being the optimal one.

(c) Assume that the cost coefficient of x_3 is modfied to $1 + \varepsilon$. State for which values of ε the current optimal basis remains being the optimal one.