

Additional exercises to Exercise 2 - Convexity
TMA947 and MMG620 Optimization, first course

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Exercise 1 (easy) Consider the polytope

$$\begin{cases} x_1 + x_2 \leq 2, \\ 2x_1 + x_2 \geq 2, \\ x_1 - x_2 \leq 1. \end{cases}$$

Use both algebraic and graphical methods to answer the following questions.

- (a) Is the point $x^1 = (1, 1)$ an extreme point?
- (b) Is the point $x^2 = \frac{1}{2}(3, 1)$ an extreme point?

Exercise 2 (medium)

- (a) Is the function $f(x_1, x_2) = x_1^2 + x_2^2 + 3x_1x_2 + 10x_1 - 11x_2 + 5$ convex?
- (b) Is the function $f(\mathbf{x}) = \sum_{i=1}^n h_i(x_i)^2$ convex, if $h_i : \mathbb{R} \rightarrow \mathbb{R}_+$ and h_i is convex?
- (c) Does b) still hold if $h_i : \mathbb{R} \rightarrow \mathbb{R}$? If not, give a counterexample.