Additional exercises to Exercise 6 - Lagrangian Duality
TMA947 and MMG620 Optimization, first course

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Exercise 1 (easy) A problem where an objective function f should be minimized has been analyzed by Lagrangian relaxing some constraints and the dual function $q(\mu)$ has been created. An iterative algorithm has produced the following results.

(i) A primal feasible x^1 has been found and $f(x^1) = 6$. μ^1 is a vector with positive Lagrangian multipliers and $q(\mu^1) = -2$. What can you say about the optimal objective value f^*?

(ii) In the next iteration, two more vectors x^2 and μ^2 (feasible) have been evaluated and $f(x^2) = 3$ and $q(\mu^2) = -4$. What can you now say about the optimal objective value f^*?

(iii) Finally, we manage to maximize the dual function and $q^* = 3$. What are the conclusions regarding f^*?

Exercise 2 (easy) Consider the problem (P):

$$\begin{aligned} \text{minimize} \quad & 4x_1 + 3x_2, \\ \text{subject to} \quad & x_1^2 + x_2^2 \leq 25, \\ & (x_1 - 4)^2 + (x_2 - 1)^2 \geq 9, \end{aligned}$$

The point $\mathbf{x_0} = (-4, -3)^T$ is a KKT to (P).

(i) Is (P) convex? Can you say anything about the optimality of $\mathbf{x_0}$?

(ii) Remove the second constraint and create a reduced problem (R). $\mathbf{x_0}$ is a KKT point in (R). What can you say about the optimality of $\mathbf{x_0}$ in (R)? In (P)?

Exercise 3 (easy) Consider the problem

$$\begin{aligned} \text{minimize} \quad & x_1^3 x_2 - x_3^2 - x_1 x_3, \\ \text{subject to} \quad & -x_1 + x_2 + x_3^2 \leq 4, \\ & x_1^3 - (x_2^2 - 1) + x_3 \geq 2, \\ & x_1 + x_3 \leq 6, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Lagrange relax the first two constraints and state the dual function q as a minimization problem.

Exercise 4 (medium) Consider the problem:

$$\begin{array}{ll} \text{minimize} & x_1 - 3x_2, \\ \text{subject to} & -x_1 + x_2 \leq 1, \\ & x_1 + x_2 \leq 4, \\ & x_1, x_2 \geq 0. \end{array}$$

Lagrangian relax the first constraint and create the Lagrange function $L(\mathbf{x}, \mu)$. Find and plot the Lagrangian dual function $q(\mu)$. What can you say about the optimal value of the primal problem? Can you find the optimal solution x^* to the primal problem?

Exercise 5 (medium) Consider the problem:

$$\begin{array}{ll} \text{minimize} & -2x_1 + x_2, \\ \text{subject to} & x_1 + x_2 \leq 3, \\ & \mathbf{x} \in \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}. \end{array}$$

Lagrange relax the inequality constraint. Which of the optimality conditions can not be fulfilled? Calculate the optimality gap $\Gamma = f^* - q^*$.