Additional exercises to Exercise 7 - Geomteric LP TMA947 and MMG620 Optimization, first course

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Exercise 1 (medium) Consider the feasible set $\{x \in \mathbb{R}^n | Ax \leq b\, x \geq 0\}$, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}, b = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}.$$

Draw the feasible region (preferably in MATLAB[1]). Write the program on standard form and find a BFS corresponding to the extreme point (1,1,1). Is it degenerate? How many different BFS correspond to this point? Compare to the extreme point (1,2,0).

Exercise 2 (easy) Write on standard form

$$\begin{array}{ll} \text{maximize} & 3x_1 - 6x_2, \\ \text{subject to} & 10x_1 - 3x_2 = 5, \\ & -x_1 - 3x_2 \geq 7, \\ & x_2 \geq 5. \end{array}$$

Exercise 3 (easy) Consider the polyhedron

$$\begin{aligned} x_1 + x_2 &\geq 1, \\ x_1 - x_2 &\leq 1, \\ -x_1 + x_2 &\leq 1, \\ x_1 &\leq 2, \\ x_2 &\leq 2. \end{aligned}$$

Find the BFS which corresponds to the extreme point (2,2). Construct new basic solutions by using four out of the five columns included in the BFS corresponding to (2,2) and one column previously not included. Can you obtain any BFS? Which ones? What does theory say about this? (Hint: use MATLAB or Mathematica to calculate $B^{-1}b$. Note also that a variable has to be included into the basis in order to obtain a non-zero value.)

[1] Introduce an indicator function χ for the polyhedron such that $\chi(x) = 1$ for $x \in P$ and $\chi(x) = 0$ otherwise. Use the command isosurface. Do not use to many gridpoints!

Exercise 4 (easy) Solve the following LP grahically.

$$\begin{aligned} \text{minimize} \quad & x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 4 \\ & x_1 + x_2 \geq 2 \\ & x_1 + 2x_2 \geq 3 \\ & x_1, 2x_2 \leq 0. \end{aligned}$$

Is the optimal solution a BFS, if so, is it unique?