

Additional exercises to Exercise 8 - the simplex method TMA947 and MMG620 Optimization, first course

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Exercise 1 (easy) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{ll} \text{minimize} & z = -2x_1 + x_2 \\ \text{subject to} & x_1 - 3x_2 \leq -3, \\ & 0 \leq x_1, \\ & 0 \leq x_2 \leq 2. \end{array}$$

Exercise 2 (medium) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{ll} \text{minimize} & z = x_1 + 2x_2 \\ \text{subject to} & 2x_1 - 2x_2 \leq -2, \\ & 2x_1 + x_2 \leq 2, \\ & x_1 \in \mathbb{R}, \\ & x_2 \geq 0. \end{array}$$

Exercise 3 (medium) Consider the application of the simplex method to a general LP and suppose that you, unlike in the standard procedure taught in this course, at some iteration *a*) choose the entering variable to be a non-basic variable with a negative reduced cost but not having the most negative reduced cost, or *b*) choose the outgoing variable as a basic variable with the $B^{-1}N_{j^*}$ component > 0 but not fulfilling the minimum ratio test. Which of these choices is a critical mistake?

Exercise 4 (easy) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{ll} \text{minimize} & z = 2x_1 - x_2 + x_3, \\ \text{subject to} & x_1 + 2x_2 - x_3 \leq 7, \\ & -2x_1 + x_2 - 3x_3 \leq -3, \\ & x_1, \quad x_2, \quad x_3 \geq 0. \end{array}$$

Exercise 5 (easy) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{aligned}
 &\text{minimize} && z = -x_1 + x_2, \\
 &\text{subject to} && -x_1 + 2x_2 \geq 1/2, \\
 &&& -2x_1 - 2x_2 \geq 1, \\
 &&& x_1 \in \mathbb{R} \text{ (free)}, \\
 &&& x_2 \geq 0.
 \end{aligned}$$

Exercise 6 (medium) Solve the following linear program using phase I and II of the simplex method.

$$\begin{aligned}
 &\text{minimize} && z = 2x_1 \\
 &\text{subject to} && x_1 - x_3 = 3, \\
 &&& x_1 - x_2 - 2x_4 = 1, \\
 &&& 2x_1 + x_4 \leq 7, \\
 &&& x_1, x_2, x_3, x_4 \geq 0.
 \end{aligned}$$

Exercise 7 (easy) Solve the following linear program using phase I and II of the simplex method.

$$\begin{aligned}
 &\text{minimize} && z = x_1 + x_2 + 3x_3, \\
 &\text{subject to} && -x_2 + 3x_3 \leq -1, \\
 &&& -2x_1 + x_2 - x_3 \leq 1, \\
 &&& x_1, x_2, x_3 \geq 0.
 \end{aligned}$$

Is the optimal solution unique?

Exercise 8 (medium) Solve the following linear program:

$$\begin{aligned}
 &\text{minimize} && z = -x_1 - x_2, \\
 &\text{subject to} && -x_1 - 2x_2 - x_3 = 2, \\
 &&& 3x_1 + x_2 \leq -1, \\
 &&& x_2, x_3 \geq 0, \\
 &&& x_1 \in \mathbb{R} \text{ (free)}.
 \end{aligned}$$

Is the optimal solution unique?