Additional exercises to Exercise 9 - LP Duality TMA947 and MMG620 Optimization, first course

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Exercise 1 (easy) Formulate the dual to the following problem

$$\begin{array}{ll} \text{maximize} & 3x_1 + 2x_2 \\ \text{subject to} & x_1 + 2x_2 \leq 3 \\ & x_1 + x_2 \leq 10 \\ & 5x_1 - x_2 \geq 8 \\ & x_1 \geq 0, \quad x_2 \leq 0. \end{array}$$

Exercise 2 (easy) Consider the following LP problem

$$\begin{array}{ll} \text{minimize} & 9x_1 + 3x_2 + 2x_3 + 2x_2 \\ \text{subject to} & \sum_{i=1}^{4} x_i \geq 1, \\ & 3x_1 - x_2 + 2x_4 \geq 1, \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \leq 0. \end{array}$$

(a) Use LP duality and a graphicall solution to obtain the optimal objective value z^*.

(b) Use complementary slackness to obtain the optimal solution x^*.

Exercise 3 (medium) Assume the following constraint matrix

$$A = \begin{pmatrix} \cdots a_1^T \cdots \\ \cdots a_2^T \cdots \\ \vdots \\ \cdots a_n^T \cdots \end{pmatrix}.$$

Consider the relaxation of a standard LP problem $\min\{c^T x | Ax \geq b, x \geq 0\}$, where we allow a violation of the constraints, but bound the sum of violations by epsilon.

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & a_i^T x \geq b_i - v_i,\ i = 1, \ldots, n, \end{array}$$

$$\sum_{i=1}^{n} v_i \leq \varepsilon,$$
$$x \geq 0, v \geq 0.$$

Formulate the dual and give it an interpretation.

Exercise 4 (medium)

(a) If an LP primal is infeasible, what can you say about its LP dual?

(b) If the LP primal has an optimal solution with reduced costs strictly greater than zero. What can you say about its LP dual?

(c) If the LP dual is unbounded, what can you say about the LP primal?

(d) According to theorem 10.12 an optimal primal dual pair must satisfy primal feasiblity, dual feasibility and complementarity. Which of these conditions is satisfied during the iteration of the simplex algorithm?