Chalmers University of Technology/Gothenburg University — Optimization
Mathematical Sciences — Course information 2012
Michael Patriksson — October 25, 2012

TMA947 Nonlinear optimisation, 7.5 credits
MMG621 Nonlinear optimisation, 7.5 credits

The purpose of this basic course in optimization is to provide

(I) knowledge of some important classes of optimization problems and of application areas of optimization modelling and methods;

(II) practice in describing relevant parts of a real-world problem in a mathematical optimization model;

(III) an understanding of necessary and sufficient optimality criteria, of their consequences, and of the basic mathematical theory upon which they are built;

(IV) examples of optimization algorithms that are naturally developed from this theory, their convergence analysis, and their application to practical optimization problems.

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Course presentation

CONTENTS: The main focus of the course is on optimization problems in continuous variables; it builds a foundation for the analysis of an optimization problem. We can roughly separate the material into the following areas:

Convex analysis: convex set, polytope, polyhedron, cone, representation theorem, extreme point, separation theorem, Farkas Lemma, convex function

Optimality conditions and duality: global/local optimum, existence and uniqueness of optimal solutions, variational inequality, Karush–Kuhn–Tucker (KKT) conditions, complementarity conditions, Lagrange multiplier, Lagrangian dual problem, global optimality conditions, weak/strong duality

Linear programming (LP): LP models, LP algebra and geometry, basic feasible solution (BFS), the Simplex method, termination, LP duality, optimality conditions, strong duality, complementarity, interior point methods, sensitivity analysis

Convex optimization: convex optimization problems, semi-definite programming,

Nonlinear optimization methods: direction of descent, line search, (quasi-)Newton method, Frank–Wolfe method, gradient projection, exterior and interior penalty, sequential quadratic programming

We also touch upon other important problem areas within optimization, such as integer programming and network optimization.

PREREQUISITES: Passed courses on analysis (in one and several variables) and linear algebra; familiarity with matrix/vector notation and calculus, differential calculus. Reading Chapter 2 in the book (i) below provides a partial background, especially to the mathematical notation used and most of the important basic mathematical terminology.

ORGANIZATION: Lectures, exercises, a project assignment, and computer exercises.

COURSE LITERATURE:

(i) *An Introduction to Optimization* by N. Andréasson, A. Evgrafov, and M. Patriksson, published by Studentlitteratur in 2005 and found in the Cremona book store

(ii) Hand-outs from books and articles

COURSE REQUIREMENTS: The course content is defined by the literature references in the plan below.

EXAMINATION:

- Written exam (first opportunity 17/12, 8.30–13.30, V building)—gives 6 credits
- Project assignment—gives 1.5 credits
- Two correctly solved computer exercises

BONUS SYSTEM:

- Active participation during *exercises* gives at most 2 bonus points on the *first* exam only
- Active participation during *master classes* gives at most 2 bonus points towards achieving grade 4/5 (Chalmers) and grade VG (GU) on the *first* exam only

SCHEDULE:

Lectures: on Tuesdays 13.15–15.00 and Fridays 8.00–9.45. *Exceptions: Lecture 2 follows immediately after Lecture 1, on 2/11 10.00–11.45.* Lectures are given in English. For locations, see the schedule below.

Exercises: on Tuesdays 15.15–17.00 and Fridays 10.00–11.45 in two parallel groups: (I) exercises in Swedish (Emil/Magnus); (II) exercises in English (Zuzana). *Exception both for (I) and (II): no exercise 2/11 (see above).* For locations, see the schedule below.

Project: Teachers are available for questions in the computer rooms, which are also booked for work on the project, on 29/11 (room: MV:F25) at 15.15–19.00. (Presence is not obligatory.) At other times, work is done individually. Deadline for handing in the project model: 15/11. Hand-out of correct AMPL model: 28/11. Deadline for handing in the project report: 5/12.

Computer exercises: The computer exercise are scheduled to take place when also teachers are available, on 22/11 and 6/12, respectively (room booked: MV:F25), and on both occasions at 15.15–19.00. (Presence is not obligatory.) The computer exercises can be performed individually, but preferably in groups of two (and *strictly not* more than two). Deadline for handing in the report, unless passed through oral examination on site during the scheduled sessions: one week following each computer exercise.

Important note: The computer exercises *require* at least one hour of preparation each; having done that preparation, two–three hours should be enough to complete an exercise by the computer.

Information about the project and computer exercises are found on the web page `https://pingpong.chalmers.se/courseId/1953/`.

This course information, the course literature, project and computer exercise materials, most hand-outs and previous exams will also be found on this page.

COURSE PLAN, LECTURES:

Week 1

Le 1 **(2/11)** [Euler, Physics building] *Course presentation.* Subject description. Course map. Applications.
Optimization modelling. Modelling. Problem analysis. Classification.
(i): Chapter 1, 2

Le 2 **(2/11)** [Euler, Physics building] *Convexity.* Convex sets and functions. Polyhedra. The Representation Theorem. Separation. Farkas Lemma.
(i): Chapter 3

Week 2

Le 3 **(6/11)** [Euler, Physics building] *Optimality conditions, introduction.* Local and global optimality. Existence of optimal solutions. Feasible directions. Necessary and sufficient conditions for local or global optimality when the feasible set is convex.
(i): Chapter 4.1–4.3

Le 4 **(9/11)** [Euler, Physics building] *Unconstrained optimization methods.* Search directions. Line searches. Termination criteria. Steepest descent. Derivative-free methods.
(i): Chapter 11
(ii): Material on derivative-free optimization

Week 3

Le 5 **(13/11)** [Euler, Physics building] *Optimality conditions, continued.* Necessary and sufficient conditions for local or global optimality when the feasible set is convex, continued.
The Karush–Kuhn–Tucker conditions. Introduction to the primal–dual optimality conditions (KKT).
(i): Chapter 4.4–, 5.1–5.4

Le 6 **(16/11)** [Euler, Physics buildning] *The Karush–Kuhn–Tucker conditions, continued.* Constraint qualifications. The Fritz–John conditions. The Karush–Kuhn–Tucker conditions: necessary and sufficient conditions for local or global optimality.
(i): Chapter 5

Le 7 **(20/11)** [Euler, Physics building] *Convex duality*. The Lagrangian dual problem. Weak and strong duality. Getting the primal solution. **Week 4**
(i): Chapter 6

Le 8 **(23/11)** [Euler, Physics building] *Linear programming*. Introduction to linear programming. Modelling. Basic feasible solutions and extreme points (algebra versus geometry in linear programming). The simplex method, introduction.
(i): Chapter 7, 8

Le 9 **(27/11)** [Euler, Physics building] *Linear programming, continued*. The Simplex method. Degeneracy. Termination. Complexity. Duality. **Week 5**
(i): Chapter 9, 10

Le 10 **(30/11)** [Euler, Physics building] *Convex optimization*. Semi-definite programming. Subgradient optimization. Algorithms.
(i): Hand-outs

Le 11 **(4/12)** [Euler, Physics building] *Integer programming*. Modelling. Applications. Algorithms. **Week 6**
(i): Hand-outs

Le 12 **(7/12)** [Euler, Physics building] *Nonlinear optimization methods: convex feasible sets*. The gradient projection method. The Frank–Wolfe method. Simplicial decomposition. Applications.
(i): Chapter 12, 6.3

Le 13 **(11/12)** [Euler, Physics building] *Nonlinear optimization methods: general sets*. Penalty and barrier methods. Interior point methods for linear programming, orientation. Sequential quadratic programming. **Week 7**
(i): Chapter 13

Le 14 **(14/12)** [Euler, Physics building] *An overview of the course*. Questions. Old exams.

COURSE PLAN, EXERCISES:

<u>**Ex 1**</u> **(6/11)** [MV:F26,33] Modelling. **Week 2**
(i): Chapter 1

<u>**Ex 2**</u> **(9/11)** [MV:F31,33] Convexity. Polyhedra. Separation. Optimality.
(i): Chapter 3

<u>**Ex 3**</u> **(13/11)** [MV:F26,33] Local and global minimum. Feasible sets. Optimality conditions. Weierstrass' Theorem **Week 3**
(i): Chapter 4

<u>**Ex 4**</u> **(16/11)** [MV:F31,33] Unconstrained optimization.
(i): Chapter 11

<u>**Ex 5**</u> **(20/11)** [MV:F26,33] The KKT conditions. **Week 4**
(i): Chapter 5

<u>**Ex 6**</u> **(23/11)** [MV:F31,33] Lagrangian duality.
(i): Chapter 6

<u>**Ex 7**</u> **(27/11)** [MV:F26,33] Geometry of LPs. **Week 5**
(i): Chapters 7, 8

<u>**Ex 8**</u> **(30/11)** [MV:F31,33] The Simplex method. Duality.
(i): Chapter 9, 10

<u>**Ex 9**</u> **(4/12)** [MV:F26,33] Sensitivity analysis. **Week 6**
(i): Chapter 10

<u>**Ex 10**</u> **(7/12)** [MV:F31,33] Convex optimization.
(i): Hand-outs

<u>**Ex 11**</u> **(14/12)** [MV:F26,33] Integer programming. **Week 7**
(i): Hand-outs

<u>**Ex 12**</u> **(11/12)** [MV:F31,33] Algorithms for convexly constrained optimization. The Frank–Wolfe and simplicial decomposition algorithms. **Week 7**
(i): Chapter 12