TMA947 / MMG621 — Nonlinear optimisation

Exercise 4 – Linear programming

October 23, 2017

E4.1 (medium) Consider the feasible set $\{\boldsymbol{x} \in \mathbb{R}^n | A\boldsymbol{x} \leq \boldsymbol{b},\ x \geq \boldsymbol{0}^n\}$, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}.$$

Draw the feasible region (preferably in MATLAB[1]). Write the program on standard form and find a BFS corresponding to the extreme point $(1, 1, 1)^T$. Is it degenerate? How many different BFS correspond to this point? Compare to the extreme point $(1, 2, 0)^T$.

E4.2 (easy) Write on standard form

$$\begin{aligned} \text{maximize} \quad & 3x_1 - 6x_2, \\ \text{subject to} \quad & 10x_1 - 3x_2 = 5, \\ & -x_1 - 3x_2 \geq 7, \\ & x_2 \geq 5. \end{aligned}$$

E4.3 (easy) Consider the polyhedron

$$\begin{aligned} x_1 + x_2 &\geq 1, \\ x_1 - x_2 &\leq 1, \\ -x_1 + x_2 &\leq 1, \\ x_1 &\leq 2, \\ x_2 &\leq 2. \end{aligned}$$

Find the BFS which corresponds to the extreme point $(2, 2)^T$. Construct new basic solutions by using four out of the five columns included in the BFS corresponding to $(2, 2)^T$ and one column previously not included. Can you obtain any BFS? Which ones? What does theory say about this? (Hint: use MATLAB or Mathematica to calculate $B^{-1}\boldsymbol{b}$. Note also that a variable has to be included into the basis in order to obtain a non-zero value.)

E4.4 (easy) Solve the following LP grahically.

$$\begin{aligned} \text{minimize} \quad & x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 4 \\ & x_1 + x_2 \geq 2 \\ & x_1 + 2x_2 \geq 3 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Is the optimal solution a BFS, if so, is it unique?

[1] Introduce an indicator function χ for the polyhedron such that $\chi(\boldsymbol{x}) = 1$ for $\boldsymbol{x} \in P$ and $\chi(\boldsymbol{x}) = 0$ otherwise. Use the command isosurface. Do not use to many gridpoints!

E4.5 (easy) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{lrl} \text{minimize} & z = -2x_1 + x_2 & \\ \text{subject to} & x_1 - 3x_2 & \leq -3, \\ & 0 \leq x_1, & \\ & 0 \leq x_2 \leq & 2. \end{array}$$

E4.6 (medium) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{lrl} \text{minimize} & z = x_1 + 2x_2 & \\ \text{subject to} & 2x_1 - 2x_2 & \leq -2, \\ & 2x_1 + x_2 & \leq 2, \\ & x_1 & \in \mathbb{R}, \\ & x_2 & \geq 0. \end{array}$$

E4.7 (medium) Consider the application of the simplex method to a general LP and suppose that you, unlike in the standard procedure taught in this course, at some iteration *a*) choose the entering variable to be a non-basic variable with a negative reduced cost but not having the most negative reduced cost, or *b*) choose the outgoing variable as a basic variable with the $B^{-1}N_{j^*}$ component > 0 but not fulfilling the minimum ratio test. Which of these choices is a critical mistake?

E4.8 (easy) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{lrl} \text{minimize} & z = 2x_1 - x_2 + x_3, & \\ \text{subject to} & x_1 + 2x_2 - x_3 & \leq 7, \\ & -2x_1 + x_2 - 3x_3 & \leq -3, \\ & x_1, \quad x_2, \quad x_3 & \geq 0. \end{array}$$

E4.9 (easy) Solve the following linear program using Phase I and II of the simplex method.

$$\begin{array}{lrl} \text{minimize} & z = -x_1 + x_2, & \\ \text{subject to} & -x_1 + 2x_2 & \geq 1/2, \\ & -2x_1 - 2x_2 & \geq 1, \\ & x_1 & \in \mathbb{R} \text{ (free)}, \\ & x_2 & \geq 0. \end{array}$$

E4.10 (medium) Solve the following linear program using phase I and II of the simplex method.

$$\begin{array}{lrl} \text{minimize} & z = 2x_1 & \\ \text{subject to} & x_1 - x_3 & = 3, \\ & x_1 - x_2 - 2x_4 & = 1, \\ & 2x_1 + x_4 & \leq 7, \\ & x_1, \quad x_2, \quad x_3, \quad x_4 & \geq 0. \end{array}$$

E4.11 (easy) Solve the following linear program using phase I and II of the simplex method.

$$\begin{aligned} \text{minimize} \quad & z = x_1 + x_2 + 3x_3, \\ \text{subject to} \quad & - x_2 + 3x_3 \leq -1, \\ & -2x_1 + x_2 - x_3 \leq 1, \\ & x_1, \; x_2, \; x_3 \geq 0. \end{aligned}$$

Is the optimal solution unique?

E4.12 (medium) Solve the following linear program:

$$\begin{aligned} \text{minimize} \quad & z = -x_1 - x_2, \\ \text{subject to} \quad & -x_1 - 2x_2 - x_3 = 2, \\ & 3x_1 + x_2 \leq -1, \\ & x_2, \; x_3 \geq 0, \\ & x_1 \in \mathbb{R} \text{ (free)}. \end{aligned}$$

Is the optimal solution unique?