TMA947 / MMG621 — Nonlinear optimisation

# Exercise 6 – Integer programming and Feasible direction methods

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**E6.1 (easy)** (Exercise 13.5, *Optimization, Exercises*, Henningsson, Lundgren, Rönnqvist, Värbrand, Studentlitteratur, 2010)
A basketball coach is to announce the start lineup (five players) for tonight's game. She has seven players to choose from and she has assessed each player's performance in four important activities; passing, shots, rebounds, and defense (1=bad, 2=good, 3=excellent). The assessment is given in the table below. Here it is also given information on which position each player can have (G=guard, F=forward, C=center). The start lineup must satisfy the following constraints.

- At least three players must be able to act as guard, at least two must be able to play as forward, and at least one must be able to play as center.
- Average performance in the activities passing, shots and returns must be at least 2.
- If player No 3 starts, then player No 6 cannot start.
- If player No 1 starts, then both player No 4 and No 5 must start.
- Either of player No 2 or player No 3 must start.

The coach wants that the defense is as strong as possible. Formulate the coach's problem as an integer linear optimization problem.

Table 1: The player assessments

| Player | Position | Passing | Shots | Rebounds | Defense |
|---|---|---|---|---|---|
| 1 | G | 3 | 3 | 1 | 3 |
| 2 | C | 2 | 1 | 3 | 2 |
| 3 | G, F | 2 | 3 | 2 | 2 |
| 4 | F, C | 1 | 3 | 3 | 1 |
| 5 | G, F | 1 | 3 | 1 | 2 |
| 6 | F, C | 3 | 1 | 2 | 3 |
| 7 | G, F | 3 | 2 | 2 | 1 |

**E6.2 (medium)** Formulate the following constraints as linear constraints by introducing binary variables and possible some large number $M$. Each part a)–c) is a separate problem.

a) The variable $x$ can only take the values $1, 6$ or $8$.

b) At least two of the following three constraints must be satisfied.

$$\begin{aligned} x_1 + \phantom{2}x_2 &\leq 6, \\ 2x_1 - 3x_2 &\leq 4, \\ x_1 + 2x_2 &\geq 1. \end{aligned}$$

c) The two vectors of binary variables $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are not identical. (E.g., $\boldsymbol{x} = (1, 1, 0)$ and $\boldsymbol{y} = (1, 1, 1)$ are feasible vectors, but $\boldsymbol{x} = (1, 1, 1)$ and $\boldsymbol{y} = (1, 1, 1)$ are not.)

**E6.3 (medium)** Consider the problem to

$$\begin{aligned} \text{minimize} \quad & -x_1 - \phantom{12}x_2 \\ \text{subject to} \quad & 11x_1 - \phantom{1}8x_2 \leq 22, \\ & 11x_1 - 12x_2 \geq 0, \\ & \phantom{11}x_1 \quad\;\; x_2 \geq 0, \text{ integer.} \end{aligned}$$

Solve the problem with and without the integer restrictions on the variables. Comment on the solutions.

**E6.4 (medium)** Consider the problem to

$$\begin{aligned} \text{minimize} \quad & -4x_1 \;\; -x_2 \\ \text{subject to} \quad & \phantom{-}3x_1 + \;\; x_2 \leq 8, \\ & \phantom{-3x_1 +}\;\; x_2 \leq 2, \\ & \phantom{-3}x_1 \quad\;\; x_2 \geq 0, \text{ integer.} \end{aligned}$$

Solve the problem with and without the integer restrictions on the variables. Comment on the solutions. What happens if we add the constraint $x_1 \leq 2$?

**E6.5 (medium)** Consider the problem to

$$\begin{aligned} \text{minimize} \quad & (x_1 - 1)^2 + \left(x_2 - \frac{1}{2}\right)^2, \\ \text{subject to} \quad & 0 \leq x_1 \leq 1, \\ & 0 \leq x_2 \leq 1. \end{aligned}$$

Draw the first five iterates obtained using the Frank-Wolfe algorithm starting in $\boldsymbol{x}_0 = (0,0)^{\mathrm{T}}$. Comment on the behavior of the algorithm. (Note: You do not have to perform the calculations, just draw the iterates approximately using your knowledge of how the algorithm works)

**E6.6 (medium)** Consider again the problem stated in exercise **E6.5**. Draw the first five iterates obtained using the simplicial decomposition algorithm starting in $\boldsymbol{x}_0 = (0,0)^{\mathrm{T}}$. Comment on the difference compared to the Frank-Wolfe algorithm. (Note: You do not have to perform the calculations, just draw the iterates approximately using your knowledge of how the algorithm works)