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Mathematical Sciences	MMG621

Planning of the natural gas transmission, part I: Modelling

October 10, 2016

1 Introduction

The background to this project is a study of the gas network in Belgium performed in 1989 by the national federation of the gas industry. All numerical data comes from the 1989 statistical yearbook of the federation (Figaz 1990). The material has been adapted to a smaller project by aggregating the original data.

Belgium has no domestic gas resources and imports all of its natural gas from Algeria, and Norway. In the problem considered (which no longer corresponds to the current situation), the Algerian gas is delivered at the Zeebrugge terminal. The gas from Norway is piped through the Netherlands and crosses the Belgian border at Voeren.

In this assignment we consider a situation where the gas merchant and transmission functions are integrated in a single company referred to as the gas company operating a transmission network. The company must decide on the quantities of gas to buy from several sources in order to satisfy the demand distributed over different nodes at some minimal guaranteed pressure with the aim to minimize the total supply cost of a gas transmission.

The following sections give a short description of gas transmission. After this follows a short description of the different demand regions, the contracted gas supplies, and the pipelines used in Belgium. The description includes prices, minimal and maximal quantities and pressures, demand data, and data regarding the pipeline flows.

2 Gas transmission

The network of a gas company consists of several supply nodes where the gas is injected into the system, several demand nodes where the gas flows out of the system, and other intermediate nodes where the gas is rerouted. At each node the gas pressure is measured. Pipelines are represented by arcs linking the nodes. There are two types of arcs: *passive arcs* correspond to pipelines, and *active arcs* correspond to pipelines with a compressor. In this assignment we have eliminated the smaller demand nodes and some intermediate nodes. We are left with two supply nodes, three intermediate nodes, and seven demand nodes, see Figure 1.

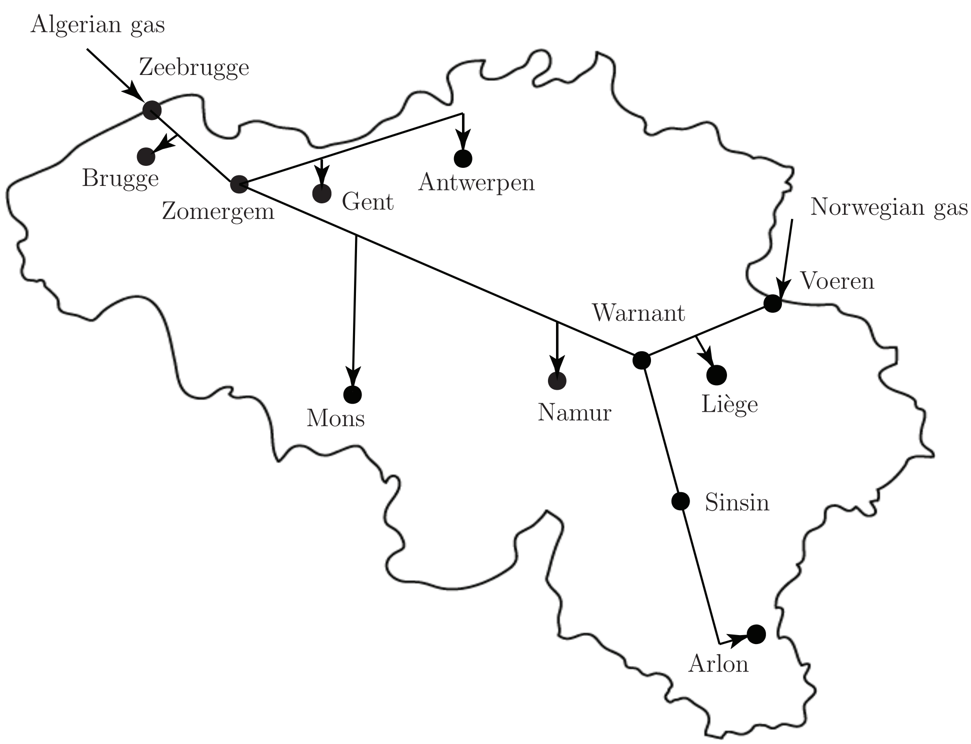

Figure 1: Belgium gas network

3 Nodes

At a supply node, the gas inflow must remain within take limitations specified in the contracts. A gas contract specifies an average daily quantity to be taken by the transmission company from the producer. Depending on the flexibility of the contracts, the transmission company has the possibility of lifting a quantity ranging between a lower and an upper bounds on the contracted quantity.

At a demand node, the gas outflow must be greater than or equal to the demand at this node.

The gas company cannot receive the gas at a pressure higher than the one ensured by the supplier at the entry node. Conversely, at each exit node, the demand must be satisfied at a minimal pressure guaranteed to the industrial user or the local distribution company.

The flow conservation ensuring the gas balance at each node has to be satisfied.

Table 1 summarizes all the relevant information concerning the nodes considered and depicted in Figure 1.

Table 1: Nodes description

Town	Min. quantity [10^6m^3/day]	Max. quantity [10^6m^3/day]	Min. pressure [bar]	Max. pressure [bar]	Price [$/MBTU]
Zeebrugge	8.870	11.594	0.0	77.0	2.28
Brugge	$-\infty$	-3.918	30.0	80.0	0.00
Zomergem	0.0	0.0	0.0	80.0	0.00
Antwerpen	$-\infty$	-4.034	30.0	80.0	0.00
Gent	$-\infty$	-5.256	30.0	80.0	0.00
Voeren	20.344	22.012	50.0	66.2	1.68
Liège	$-\infty$	-6.365	30.0	66.2	0.00
Warnant	0.0	0.0	0.0	66.2	0.00
Namur	$-\infty$	-2.120	0.0	66.2	0.00
Mons	$-\infty$	-6.848	0.0	66.2	0.00
Sinsin	0.0	0.0	0.0	63.0	0.00
Arlon	$-\infty$	-0.222	0.0	66.2	0.00

3.1 Supply nodes

Consider the supply side of the Belgium gas market. An estimation of the daily average contracted quantity can be found in Table 2. It is assumed that the daily takes can range between the minimal and the maximal contracted quantities. The gas company aims to minimize the total cost of the supplies. The purchase price of the gas delivered is given in Table 2 in dollars per million British thermal units [$/MBTU].

Table 2: Minimal and maximal daily quantities

Producer	Daily quantity [10^6m^3/day]	Min. quantity [10^6m^3/day]	Max. quantity [10^6m^3/day]	Price [$/MBTU]
Norway	21.540	20.344	22.012	1.68
Algeria	10.082	8.870	11.594	2.28

3.2 Demand nodes

The total demand of each Belgian province considered has been assigned to the main town of the province and can be found in Table 3.

Table 3: Daily demand by province

Province	Demand $[10^6 \mathrm{m}^3/\mathrm{day}]$
Antwerpen	4.034
Arlon	0.222
Brugge	3.918
Gent	5.256
Liège	6.365
Mons	6.848
Namur	2.120

4 Pipelines

Pipelines are represented by arcs linking the nodes. Note that the arc flow direction depends on the pressures at the end nodes. We distinguish between the passive and active arcs. For a passive arc, the relation between the flow f_{ij} in the arc (i, j) and the pressures p_i and p_j in the nodes i and j is of the following form

$$\mathrm{sign}(f_{ij}) f_{ij}^2 = C_{ij}^2 (p_i^2 - p_j^2),$$

where C_{ij} is a constant that depends on the length, the diameter, the absolute rugosity of pipe, and the gas composition. For each pipeline in the network, we compute the term C_{ij}^2 by the following formula:

$$C_{ij}^2 = 96.074830 \cdot 10^{-15} \frac{D_{ij}^5}{\lambda_{ij} z T L_{ij} \delta},$$

where

$$\frac{1}{\lambda_{ij}} = \left[2 \log_{10} \left(\frac{3.7 D_{ij}}{\varepsilon} \right) \right]^2,$$

where L_{ij} [km] is the length of the pipe, D_{ij} [mm] is the interior diameter of the pipe, $T = 281.15$ K is the gas temperature, $\varepsilon = 0.05$ mm is the absolute pipe roughness, $\delta = 0.6106$ is the density of the gas relative to air, and $z = 0.8$ is the gas compressibility factor. The length and the interior diameter are given in Table 4 for each arc. The last column refers to the type of arc. The only compressor is located at Sinsin.

For an active arc corresponding to a pipeline with a compressor, which increases the pressure along the pipe, the relation between the flow and the corresponding pressures is

$$\mathrm{sign}(f_{ij}) f_{ij}^2 \geq C_{ij}^2 (p_i^2 - p_j^2).$$

There is an upper bound on the pressure at the exit of the compressor for each active arc. We do not consider any cost associated with the pressure increase. For active arcs, the direction of the flow f_{ij} is fixed to be from node i to node j.

Table 4: Pipeline description

From	To	Diameter [mm]	Length [km]	Pipe type
Zeebrugge	Brugge	890.0	12	passive
Brugge	Zomergem	890.0	26	passive
Antwerpen	Gent	590.1	39	passive
Gent	Zomergem	590.1	14	passive
Zomergem	Mons	890.0	75	passive
Mons	Namur	890.0	55	passive
Namur	Warnant	890.0	42	passive
Voeren	Liège	890.0	22	passive
Liège	Warnant	590.1	25	passive
Warnant	Sinsin	315.5	40	active
Sinsin	Arlon	315.5	98	passive

5 Assignment

Your task is to write a report that describes a nonlinear programming model for minimizing the total cost of supplying the demand of gas distributed over different nodes at some minimal guaranteed pressure in Belgium. The nonlinear program should be based on the data given in the Sections 1–4.

To make the model easy to understand, even for people with just a basic knowledge of mathematics, the variables, the objective function, and the constraints of the model have to be clearly defined and explained.

In the industry it is more likely that your model will be used if the managers understand it. One of the purposes of this assignment is that you should learn how to formulate a complex problem as an easily understandable nonlinear program. Therefore, we have the following requirement on your report:

- it shall include a figure illustrating the gas flows in the problem;
- the variables must be clearly defined and connected to the figure;
- the objective function and the constraints should be clearly described;
- it must be written with a text formatting tool (e.g., LaTeX or Word); and
- it should look professional!

Since there is a lot of data in the problem you do not need to give explicit values of the coefficients in the model; on the contrary, we encourage the use of general notation, such as L_{ij} for the length of the pipeline from node i to node j. However, every coefficient introduced must be clearly defined!

The report should be written in groups of two (preferable) or individually. Names and personal code numbers of the group members as well as the e-mail addresses should be given on the front page of the report. You may discuss the problem with other students. However, each group must hand in their own solution. The report will be checked for plagiarism via `http://www.urkund.com`. **The deadline for handing in the model report through PingPong is November 21!**

To the groups that have delivered an accepted model report, we will later distribute a version of the model, implemented in AMPL, which will be used to answer question in the second part of the project.

Good luck!