TMA 965 : Discrete Matematics
D3-spring, 03

Homework 4

The following rules are in force for all homework exercises. (see the course description):

1. Explain (prove) all of your answers.
2. Inform me whom you have worked with. (Recall you are encouraged to cooperate with other students to figure out how to solve the problems but the final formulating and writing up of the solutions must be done yourself.)
3. Save (of course) all your homeworks together with my comments.
4. All exercises given refer to the new version of the book which arrived at Cremona on Wednesday. **************

Warm up problems

[Although I call them "warm up" problems, they are not necessarily always much easier than the exercises to be handed in. The point is to give you more exercises to practice on so that you can understand the material better.]

1. Exercise 13.1.1

2. Exercise 13.1.2

3. What is the last digit in the number 7^{93}?

4 Exercise 13.6.7.

5 Exercise 13.2.3. Thinking about linear algebra, how might you have predicted that there would be a solution in $\frac{Z}{7}$ but perhaps not in $\frac{Z}{5}$?

6 Consider the collection of two simultaneous congruences modulo n where $n = 5, 7, 10$.
$x + 4y \equiv -1$
$3x + 2y \equiv 2$
Solve these two simultaneous congruences for all the cases $n = 5, 7, 10$.

When $n = 10$, we can multiple the second equation by -2 and add it to the first giving us

$$-5x \equiv -5$$

or $x \equiv 1$. The first equation then gives $y \equiv -3$ (remembering that everything is mod(10) here). Does this give a correct solution to the case $n = 10$. If not, explain what went wrong with the computation.

To be handed in Thursday February 20

1. Exercise 13.3.7

2. Exercise 13.3.8.

3. Exercise 11.5.4.

4. Exercise 13.6.9.

5. Show that $(a + b)^p \equiv a^p + b^p (\mod p)$ for all primes p and all integers a and b. Do not use Euler's theorem but rather using the binomial formula. Is this also true if p is not prime? Prove or give a counterexample.

6. The following is an *EXTRA CREDIT* problem, is not related to the topic of this homework but has connections to some coding things that we do later on.

(IMPORTANT: Extra credit problems are to be worked on alone and not discussed with others!)

There are three of you who are working as a team and will be put in the following situation tomorrow. You will be put in a dark room and a blue or red hat will be placed on each of your heads. Blue or red each happen with probability 1/2 and independently for the 3 of you. The lights will then be turned on and you can see the other two people's hat but not your own. Each of you will then simultaneously decide to either (a) guess your hat is red (b) guess your hat is blue or (c) not make a guess about the color of your hat. The team wins if (1) everyone who does (a) or (b) guesses correctly and (2) not everyone chooses (c) (that is someone makes a real guess). Your goal is to use an algorithm or protocol which maximizes the probability of the team winning the game. You are allowed to meet before hand to discuss things and to come up with a protocol that you will then follow (but once placed in the room, no further communication is allowed). What is the maximal probability that you can achieve and what would the protocol be?