Give arguments to all solutions. List all collaborators.

Warmup

1. Biggs 10.6.1, 10.6.2
2. Biggs 12.2.5
3. Biggs 12.1.2
4. Biggs 12.3.4
5. Biggs 12.3.6
6. Biggs 12.7.1
7. A committee is to be chosen from a set of 7 woman and 4 men. How many ways are the to form the committee if
   (a) the committee has 5 people, 3 woman and 2 men?
   (b) the committee can be any size (other than empty) but it must have an equal number of men and women?
   (c) the committe has 4 people and one of them must be Mr. Smith?
   (d) the committee has 4 people, 2 of each sex, and Mr. and Ms. Smith cannot be both on the committee?
8. Let $\delta(G)$ be the smallest degree of any vertex in G. Show that G contains a path with at least $\delta(G)$ edges.
9. Biggs 17.1.2
10. Let G be a bipartite graph with an odd number of vertices. Show that G cannot have a Hamilton cycle.
11. How many Euler cycles are there in K_n (the complete graph on n vertices)? How many Hamilton cycles are there? Two cycles are considered equal if they contain the same edges.
12. Show that every tree with at least 2 vertices las at least two leaves.
13. Biggs 16.3.3

The exercises below are to be handed in Tuesday September 26, 13.15 at the latest.

1. Biggs 12.7.12 and 12.7.13
2. Let a, b and n be natural numbers. Define a relation S on the set of all integers by
   $$xSy \Leftrightarrow ax + by \equiv 0 \pmod{n} .$$
   For which (a, b) is S an equivalence relation?

3. Show that if a graph has at least two vertices, then it also has at least two vertices of the same degree.

4. Show that for any graph G holds

$$\chi(G) \leq \frac{1}{2} + \sqrt{2|E| + \frac{1}{4}} \,.$$

   Hint. Show first that $|E| \geq \binom{\chi(G)}{2}$.

5. A mouse wishes to consume a cube of cheese, consisting of $3 \times 3 \times 3$ smaller cheese cubes. Is it possible for it to start in a corner and finish in the middle, if it may only proceed from one cube to an adjacent one (that is one that shares a side)? *Hint.* Colour the cubes!

Bonus problem (no collaboration)

6. Show that the complete graph with $2n+1$ vertices, K_{2n+1}, may be viewed as the union of n Hamilton cycles on the same set of vertices, but having no edges in common.