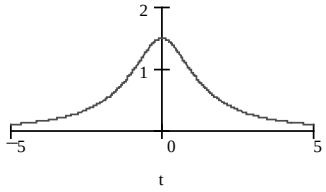
Matematiska metoder E2, fk (TMA980), delB, transformationer (98)
svar på gamla tentorna:

98-08-18

1) $x(t)=1, y(t)=e^{t}-1, z(t)=1-e^{t}\ \ (t>0),$

2a) ja **c)** $\frac{2}{3\pi}\left(2\pi-3\arctan\frac{\sqrt{3}}{2}\right)\approx 88\%,$ **b)**



e) $\frac{1}{3}\left(2e^{-|t|}-sgn\,t\,e^{-|t|}\right)$

d) $\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\,t\right),\ \left(e^{-2t}-e^{-t}+\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\,t\right)\right)\theta(t),\ \frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\,t\right)\theta(-t)-\left(e^{-2t}-e^{-t}\right)\theta(t)$

3) $\frac{1}{2}e^{-|t|}+te^{-t}\theta(t)$ **4)** $\sum_{k=1}^{\infty}\frac{1}{2}\sin\frac{k\pi}{4}-\frac{k\pi}{4}\sin\frac{3k\pi}{4}e^{-\frac{k^2\pi^2}{4}t}\sin\frac{k\pi}{4}x$

98-045-15

1) $\left(e^{\frac{1}{2}(t+1)}+(3t-4)e^{2-t}\right)\theta(t-1)+e^{\frac{1}{2}t}+2e^{-t},\ t>0$ **2)** $\frac{e^{1-j\omega}}{1+j\omega},\ \pi e^{-a},\ 0$ **3)** $2e^{t}\theta(-t-1)$

4a) ja **b)** $2y'''+5y''+6y'+2y=x'+2x$

d) $\left(-\frac{1}{6}\sin\sqrt{2}\,t-\frac{\sqrt{2}}{6}\cos\sqrt{2}\,t+\frac{A}{2}e^{-\frac{t}{2}}+\left(C\cos t+(E-D)\sin t\right)e^{-t}\right)\theta(t)\ \left(A=\frac{8\sqrt{2}}{15}, C=-\frac{\sqrt{2}}{10}, E-D=\frac{\sqrt{2}}{5}\right)$

e) $u(x,t)=\sum_{k=0}^{\infty}\frac{\vartheta}{2}(-1)^{k+1}(2k+1)\cos\frac{(2k+1)\pi}{2}x\cos(2k+1)\pi t$ (svagt)

98-045-15

1) ja **2a)** $y'''+3y''+4y'+2y=2x$ **b)** $2(1-\cos t)e^{-t}\theta(t)$ **c)** $-\frac{1}{5}\cos 2t$ **f)** $t-2$

g) $\frac{1}{2\pi}+\sum_{n=1}^{\infty}\frac{2}{\pi\sqrt{\left(n^2+1\right)\left(n^4+4\right)}}\cos\left(n\,t+\pi-\arctan\frac{n\left(n^2-4\right)}{3n^2-2}\right)$ **h)** $-3\delta(t)-\delta'(t)+5e^{t}\theta(-t)$

2) $\left(\frac{2}{\omega^2+1}-\frac{4\omega j}{\left(\omega^2+1\right)^2}\right)e^{-j\omega},\ \frac{\pi}{4e},\ \frac{3\pi}{8}$ **3)** $v(x,t)=\sum_{n=0}^{\infty}\frac{4(2n+1)}{\pi\left((2n+1)^2-400\right)}e^{-\frac{(2n+1)^2\pi^2}{4}t}\sin\frac{(2n+1)\pi}{20}x$

1997-08-19

1a) $a=1$ **c)** $-\frac{1}{8}\left(\sin\sqrt{3}t+\sqrt{3}\cos\sqrt{3}t\right)$ **d)** $-\frac{1}{8}\left(\sin\sqrt{3}t+\sqrt{3}\cos\sqrt{3}t-\sqrt{3}(1+2t)e^{-t}\right)\theta(t)$

e) $\frac{1}{2}e^{-|t|}$ **f)** $\frac{E(h_\pi)}{E(h)}=\frac{2\left(\pi+\left(1+\pi^2\right)\arctan\pi\right)}{\pi\left(1+\pi^2\right)}\ (\approx 98.8\%)$

2) $2+\sum_{n=1}^{\infty}\frac{4\sqrt{1+n^2\pi^2}}{1+4n^2\pi^2}\cos\left(2n\pi\,t-\arctan\left(4n^3\pi^3+3n\pi\right)\right)$

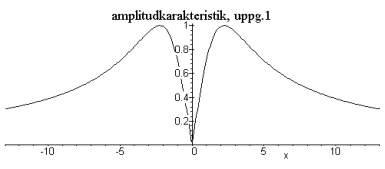
3) $u(x,t)=1+\sum_{n=0}^{\infty}\frac{32}{(2n+1)^4\pi^4}e^{-\frac{(2n+1)^2\pi^2}{4}t}\cos\frac{(2n+1)\pi}{4}x,\ \lim_{t\to\infty}u(x,t)=0$

4) $u(x,t)=\int_0^{\infty}\frac{x}{4\tau\sqrt{\pi\tau}\sqrt{1+(t-\tau)^2}}e^{-\frac{x^2}{16\tau}}d\tau$

1996-04-10

2) $\sum_{k=1}^{\infty}\frac{k\cos\left(kt+\arctan\frac{5-k^2}{4k}\right)}{\pi\sqrt{k^4+6k^2+25}}$ **3)** $\frac{2\left(e^{\pi}+t\sin\pi t-\cos\pi t\right)}{e^{\pi}\left(1+t^2\right)}$

1)



4) $\frac{\sqrt{3x}}{\sqrt{\pi t}\,t}e^{-\frac{4t^2+9x^2}{12t}}\theta(t)$ **5)** $\cos\sqrt{5}\,y_1(t-1)+\sin\sqrt{5}\,y_2(t-1)$

där $y_1(t)=\left(\sin\sqrt{5}\,t-\sqrt{5}\sin t\,e^{-2t}\right)\theta(t)$

och $y_2(t)=\left(\cos\sqrt{5}\,t+(2\sin t-\cos t)e^{-2t}\right)\theta(t)$