mat. met. E2, fk, del B, svar på gamla tentor (99)

99-08-17

1a) $\frac{1}{2}(\sinh t+\sin t)\theta(t)$ **b)** $(n+1)a^n\theta(n)$ om $a=b$, $\frac{a^{n+1}-b^{n+1}}{a-b}\theta(n)$ om $a\neq b$

2a) $h(t)=2te^{-t}\theta(t)$, tillst.ekv: $y''+2y'+y=2x$ **b)** $\sin t$ **c)** $\left(\sin t-te^{-t}\right)\theta(t)$ **d)** $\frac{1}{2}te^{-t}$

3a) $y(n-1)+3y(n)=3x(n)$ **b)** $\frac{1}{10}\left(9\cos\frac{n\pi}{2}-3\sin\frac{n\pi}{2}\right)$ **c)** $\frac{1}{10}\left(9\cos\frac{n\pi}{2}-3\sin\frac{n\pi}{2}+\left(-\frac{1}{3}\right)^n\right)\theta(n)$

4) $u(x,t)=\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}e^{-\frac{t}{2}}}{n\pi\sqrt{4n^2-1}}\sin\left(\sqrt{n^2-\frac{1}{4}}\,t\right)\sin(nx)$

99-04-07

1a) $h(t)=(\sin t+\cos t-1)e^{-t}\theta(t)$ **c)** $A(\omega)=\frac{|\omega|}{\sqrt{1+\omega^2}\sqrt{1+\omega^4}}$ **f)** $y(t)=\begin{cases}\frac{1}{2}e^t, & t\le 0\\ \left(\cos t-\sin t-\frac{1}{2}t\right)e^{-t}, & t\ge 0\end{cases}$

1d) $\frac{1}{5}\sin 2t$ **2b)** $A(\alpha)=\frac{\sqrt{2-2\cos\alpha}}{\sqrt{5+4\cos\alpha}}$ **c)** $-2^{-(n+1)}\theta(n)$ **d)** $\frac{1}{5}\left(\cos\frac{n\pi}{2}-3\sin\frac{n\pi}{2}\right)$

e) $\frac{1}{5}\left(\cos\frac{n\pi}{2}-3\sin\frac{n\pi}{2}-3(-2)^{-(n+1)}\right)\theta(n)$ **3)** $u(x,t)=\sum_{n=0}^{\infty}\frac{1}{\pi}\sin\frac{2n+1}{4}\cos(2n+1)x\,\sin(2n+1)t$

98-12-14

1b) $H(s)=\frac{1}{s^2}-\frac{2s+1}{(s+1)s^2},\hat{h}(\omega)=-\frac{1}{\omega^2}+\frac{1+2\omega j}{(1+\omega j)\omega^2}$ **c)** $\frac{3}{2}$ d) $\frac{2\pi\cos 2\pi t-\sin 2\pi t}{2\pi(1+4\pi^2)}$

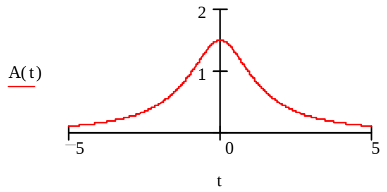
e) $y(t)=-4+\left(4-\frac{8}{3}e^{\frac{1}{2}(t-1)}-\frac{4}{3}e^{-(t-1)}\right)\theta(t-1)\ (t>0)$ **2a)** $4y(n+4)-y(n)=4(x(n+4)-x(n+2))$

b) $\frac{8|\sin\alpha|}{\sqrt{17-8\cos 4\alpha}}$ **c)** $\frac{4\sqrt{2}}{5}\sin\frac{(n+1)\pi}{4}$ **d)** $h(n)=\begin{cases}\frac{3(-1)^k-1}{2^{k+1}}, & n=2k\ge 0\\ 0, & \text{annars}\end{cases}$ **3)** $\sum_{n=0}^{\infty}\frac{2\sin\frac{(2n+1)\pi}{4}}{e^{(2n+1)\pi}}\sin\frac{(2n+1)x}{2}$

98-08-18

1) $x(t)=1, y(t)=e^t-1, z(t)=1-e^t\ \ (t>0)$,

2a) ja **c)** $\frac{2}{3\pi}\left(2\pi-3\arctan\frac{\sqrt{3}}{2}\right)\approx 88\%$, **b)**



e) $\frac{1}{3}\left(2e^{-|t|}-\operatorname{sgn} t\, e^{-|t|}\right)$

d) $\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\,t\right)$, $\left(e^{-2t}-e^{-t}+\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\,t\right)\right)\theta(t)$, $\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\,t\right)\theta(-t)-\left(e^{-2t}-e^{-t}\right)\theta(t)$

3) $\frac{1}{2}e^{-|t|}+te^{-t}\theta(t)$ **4)** $\sum_{k=1}^{\infty}\frac{1}{2}\left(\sin\left(\frac{k\pi}{4}\right)-\left(\frac{k\pi}{4}\right)^2\sin\left(\frac{3k\pi}{4}\right)\right)e^{-\frac{k^2\pi^2}{4}t}\sin\left(\frac{k\pi}{4}x\right)$

98-04-15

1) $\left(e^{\frac{1}{2}(t+1)}+(3t-4)e^{2-t}\right)\theta(t-1)+e^{\frac{1}{2}t}+2e^{-t}$, $t>0$ **2)** $\frac{e^{1-j\omega}}{1+j\omega}$, πe^{-a}, 0 **3)** $2e^t\theta(-t-1)$

4a) ja **b)** $2y'''+5y''+6y'+2y=x'+2x$ **d)** $-\frac{1}{6}\sin\sqrt{2}t-\frac{\sqrt{2}}{6}\cos\sqrt{2}t$

e) $\left(-\frac{1}{6}\sin\sqrt{2}\,t-\frac{\sqrt{2}}{6}\cos\sqrt{2}\,t+\frac{A}{2}e^{-\frac{t}{2}}+\left(C\cos t+(E-D)\sin t\right)e^{-t}\right)\theta(t)$ $\left(A=\frac{8\sqrt{2}}{15}, C=-\frac{\sqrt{2}}{10}, E-D=\frac{\sqrt{2}}{5}\right)$

5) $u(x,t)=\sum_{k=0}^{\infty}\frac{\pi}{2}(-1)^{k+1}(2k+1)\cos\frac{(2k+1)\pi}{2}x\ \cos(2k+1)\pi t$ (svagt)