Övningstenta transformer (tma980/tmv070),04-11-20, lösningar

uppgift 1

Laplacetranformation av $y'(t) - 3y(t) + 2\int_0^t y(\tau)\,d\tau = (2 - 2(t-1))\,\theta(t-1)$

ger $[y(t) = 0$ för $t < 0 \Longrightarrow y(0-) = 0]$:
$\left(s - 3 + \frac{2}{s}\right)Y(s) = \left(\frac{2}{s} - \frac{2}{s^2}\right)e^{-s} \Longrightarrow Y(s) = \frac{2(s-1)}{s(s^2-3s+2)}e^{-s} = \frac{2}{s(s-2)}e^{-s} = \left(\frac{1}{s-2} - \frac{1}{s}\right)e^{-s} \Longrightarrow \boxed{y(t) = \left(e^{2(t-1)} - 1\right)\theta(t-1)}$.

uppgift 2

$\widehat{f}(\omega) = \frac{2+2j}{\omega^2+2j} = \frac{2(1+j)}{(\omega+1-j)(\omega-(1-j))} = \frac{1+j}{1-j}\left(\frac{1}{\omega-(1-j)} - \frac{1}{\omega+1-j}\right) =$
$= \frac{1}{1-j(\omega-1)} + \frac{1}{1+j(\omega-1)}$ $\quad [\frac{1+j}{1-j} = \frac{2}{-2j}$ ...]. Detta ger nu:

a) Använd $e^{\pm jt}g(t) \supset \widehat{g}(\omega \mp 1)$ på [F17], [F18], så får du
$\boxed{f(t) = e^{jt}e^{t}\theta(-t) + e^{-jt}e^{-t}\theta(t)}$
[dela upp i Re och Im så får du $\boxed{f(t) = (\cos t - j\,\mathrm{sgn}\,t \sin t)\,e^{-|t|}}$].

b) [F12] ger: $t^2 f(t) \supset -\frac{d^2}{d\omega^2}\left(\frac{2+2j}{\omega^2+2j}\right) = (2+2j)\frac{d}{d\omega}\left(\frac{2\omega}{(\omega^2+2j)^2}\right) =$
$= 4(1+j)\frac{2j-3\omega^2}{(\omega^2+2j)^3} =: g(\omega)$, alltså
$\int_{-\infty}^{\infty} t^2 f(t)\,e^{j\sqrt{2}t}dt = g\left(-\sqrt{2}\right) = \frac{4(1+j)(2j-6)}{(2+2j)^3} = \frac{(1+j)(j-3)}{2j(1+j)} = \boxed{\frac{1+3j}{2}}$.

c) $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\widehat{f}(\omega)\,e^{j\omega t}d\omega = \frac{1+j}{\pi}\int_{-\infty}^{\infty}\frac{\omega^2-2j}{\omega^4+4}(\cos\omega t + j\sin\omega t)\,d\omega =$ [jämn-udda] $= \frac{2(1+j)}{\pi}\int_0^{\infty}\frac{\left(\omega^2-2j\right)\cos\omega t}{\omega^4+4}d\omega = \frac{2}{\pi}\int_0^{\infty}\frac{\left(\omega^2+2\right)\cos\omega t}{\omega^4+4}d\omega + j\frac{2}{\pi}\int_0^{\infty}\frac{\left(\omega^2-2\right)\cos\omega t}{\omega^4+4}d\omega$
$\Longrightarrow \int_0^{\infty}\frac{\left(\omega^2+2\right)\cos\omega t}{\omega^4+4}d\omega = \frac{\pi}{2}\,\mathrm{Re}\,f(1) = \frac{\pi}{2}\,\mathrm{Re}\left(e^{-j}e^{-1}\right) = \boxed{\frac{\pi\cos 1}{2e}}$.

uppgift 3

$$\left.\begin{array}{l} \theta(t)\sinh t = \frac{1}{2}\left(e^t - e^{-t}\right) \overset{Laplace}{\supset} \frac{1}{2}\left(\frac{1}{s-1} - \frac{1}{s+1}\right) \\ \theta(t)\cosh t = \frac{1}{2}\left(e^t + e^{-t}\right) \overset{Laplace}{\supset} \frac{1}{2}\left(\frac{1}{s-1} + \frac{1}{s+1}\right) \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow (\sinh(t)\,\theta(t)) * (\cosh(t)\,\theta(t)) \supset \frac{1}{2}\left(\frac{1}{s-1}-\frac{1}{s+1}\right)\cdot\frac{1}{2}\left(\frac{1}{s-1}+\frac{1}{s+1}\right) =$$
$$= \frac{1}{4}\left(\frac{1}{(s-1)^2}-\frac{1}{(s+1)^2}\right) \subset \frac{1}{4}\left(te^{t}-te^{-t}\right)\theta(t) = \boxed{\frac{1}{2}\sinh(t)\,\theta(t)}.$$

Du kan även multiplicera ut först:
$(\sinh(t)\,\theta(t)) * (\cosh(t)\,\theta(t)) \supset \frac{1}{(s^2-1)}\cdot\frac{s}{(s^2-1)} = -\frac{1}{2}\frac{d}{ds}\frac{1}{s^2-1} \underset{[L03]}{\subset} \frac{1}{2}\sinh(t)\,\theta(t).$