FACIT AJ

Chapter 9 Sequences, Series, and Power Series

Section 9.1 (page 526)

1. bounded, positive, increasing, convergent to 2
3. bounded, positive, convergent to 4
5. bounded below, positive, increasing, divergent to infinity
7. bounded below, positive, increasing, divergent to infinity
9. bounded, positive, decreasing, convergent to 0
11. divergent
13. divergent
15. ∞
17. 0
19. 1
21. e^{-3}
23. 0
25. 1/2
31. $\lim_{n\to\infty} a_n = 5$
33. If $\{a_n\}$ is (ultimately) decreasing, then either it is bounded below and therefore convergent, or it is unbounded below and therefore divergent to negative infinity.

Section 9.2 (page 534)

1. $\frac{1}{2}$
3. $\frac{1}{(2+\pi)^8((2+\pi)^2-1)}$
5. $\frac{25}{4\,416}$
7. $\frac{8e^4}{e-2}$

9. diverges to ∞
11. $\frac{3}{4}$
13. $\frac{1}{3}$
15. div. to ∞
17. div. to ∞
19. diverges
21. 14 m
25. If $\{a_n\}$ is ultimately negative, then the series Σa_n must either converge (if its partial sums are bounded below), or diverge to $-\infty$ (if its partial sums are not bounded below).
27. false, e.g., $\Sigma \frac{(-1)^n}{2^n}$
29. true
31. true

Section 9.3 (page 545)

1. converges
3. diverges to ∞
5. converges
7. diverges to ∞
9. converges
11. diverges to ∞
13. diverges to ∞
15. converges
17. converges
19. diverges to ∞
21. converges
23. converges

25. converges

27. $s_n + \frac{1}{3(n+1)^3} \leq s \leq s_n + \frac{1}{3n^3}; \quad n = 6$

29. $s_n + \frac{2}{\sqrt{n+1}} \leq s \leq s_n + \frac{2}{\sqrt{n}}; \quad n = 63$

31. $0 < s - s_n \leq \frac{n+2}{2^n(n+1)!(2n+3)}; \quad n = 4$

33. $0 < s - s_n \leq \frac{2^n(4n^2+6n+2)}{(2n)!(4n^2+6n)}; \quad n = 4$

39. converges, $a_n^{1/n} \to (1/e) < 1$

41. no info from ratio test, but series diverges to infinity since all terms exceed 1.

43. (b) $s \leq \frac{2}{k(1-k)}, \quad k = \frac{1}{2},$
(c) $0 < s - s_n < \frac{(1+k)^{n+1}}{2^n k(1-k)}, \quad k = \frac{n+2-\sqrt{n^2+8}}{2(n-1)}$ for $n \geq 2$

Section 9.4 (page 553)

1. conv. conditionally

3. conv. conditionally

5. diverges

7. conv. absolutely

9. conv. conditionally

11. diverges

14. 999

15. 13

17. converges absolutely if $-1 < x < 1$, conditionally if $x = -1$, diverges elsewhere

19. converges absolutely if $0 < x < 2$, conditionally if $x = 2$, diverges elsewhere

21. converges absolutely if $-2 < x < 2$, conditionally if $x = -2$, diverges elsewhere

23. converges absolutely if $-\frac{7}{2} < x < \frac{1}{2}$, conditionally if $x = -\frac{7}{2}$, diverges elsewhere

25. AST does not apply directly, but does if we remove all the 0 terms; series converges conditionally

27. (a) false, e.g., $a_n = \frac{(-1)^n}{n}$
(b) false, e.g., $a_n = \frac{\sin(n\pi/2)}{n}$ (see Exeercise 25)
(c) true

29. converges absolutely for $-1 < x < 1$, conditionally if $x = -1$, diverges elsewhere

Section 9.5 (page 564)

1. centre 0, radius 1, interval $]-1, 1[$

3. centre -2, radius 2, interval $[-4, 0[$

5. centre $\frac{3}{2}$, radius $\frac{1}{2}$, interval $]1, 2[$

7. centre 0, radius ∞, interval $]-\infty, \infty[$

9. $\frac{1}{(1-x)^3} = \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n, (-1 < x < 1)$

11. $\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1)x^n, (-1 < x < 1)$

13. $\frac{1}{(2-x)^2} = \sum_{n=0}^{\infty} \frac{n+1}{2^{n+2}} x^n, \quad (-2 < x < 2)$

15. $\ln(2-x) = \ln 2 - \sum_{n=1}^{\infty} \frac{x^n}{2^n n}, \quad (-2 \le x < 2)$

17. $\frac{1}{x^2} = \sum_{n=0}^{\infty} \frac{n+1}{2^{n+2}} (x+2)^n, \quad (-4 < x < 0)$

19. $\frac{x^3}{1-2x^2} = \sum_{n=0}^{\infty} 2^n x^{2n+3}, \quad (-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}})$

21. $(-\frac{1}{4}, \frac{1}{4})$; $\frac{1}{1+4x}$

23. $[-1, 1)$; $\frac{1}{3}$ if $x = 0$, $-\frac{1}{x^3} \ln(1-x) - \frac{1}{x^2} - \frac{1}{2x}$ otherwise

25. $(-1, 1)$; $\frac{2}{(1-x^2)^2}$

27. $3/4$

29. $\pi^2(\pi+1)/(\pi-1)^3$

31. $\ln(3/2)$

Section 9.6 (page 5752)

1. $e^{3x+1} = \sum_{n=0}^{\infty} \frac{3^n e}{n!} x^n$, (all x)

3. $\sin(x - \frac{\pi}{4}) = \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} (-1)^n [-\frac{x^{2n}}{(2n)!} + \frac{x^{2n+1}}{(2n+1)!}]$. (all x)

5. $x^2 \sin(\frac{x}{3}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{2n+1}(2n+1)!} x^{2n+3}$, (all x)

7. $\sin x \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n+1)!} x^{2n+1}$, (all x)

9. $\frac{1+x^3}{1+x^2} = 1 - x^2 + \sum_{n=2}^{\infty} (-1)^n (x^{2n-1} + x^{2n})$, $(-1 < x < 1)$

11. $\frac{1-x}{1+x} = -2 \sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$, $(-1 < x < 1)$

13. $\cosh x - \cos x = 2 \sum_{n=0}^{\infty} \frac{x^{4n+2}}{(4n+2)!}$, (all x)

15. $e^{-2x} = e^2 \sum_{n=0}^{\infty} (x+1)^n$, (all x)

17. $\cos x = \sum_{n=0}^{\infty} \frac{(1-)^{n+1}}{(2n)!} (x - \pi)^{2n}$, (all x)

19. $\ln 4 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4^n n} (x-2)^n$, $(-2 < x \leq 6)$

21. $\sin x - \cos x = \sqrt{2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x - \frac{\pi}{4})^{2n+1}$, (all x)

23. $\frac{1}{x^2} = \frac{1}{4} \sum_{n=0}^{\infty} \frac{n+1}{2^n} (x+2)^n$, $(-4 < x < 0)$

25. $(x-1) + \sum_{n=2}^{\infty} \frac{(-1)^n}{n(n-1)} (x-1)^n$ $(0 \leq x \leq 2)$

27. $1 + \frac{x^2}{2} + \frac{5x^4}{24}$

29. $x + \frac{x^2}{2} - \frac{x^3}{6}$

31. $1 + \frac{x}{2} - \frac{x^2}{8}$

33. e^{x^2} (all x)

35. $\frac{e^x - e^{-x}}{2x} = \frac{\sinh x}{x}$ if $x \neq 0$, 1 if $x = 0$

37. (a) $1 + x + x^2$, (b) $3 + 3(x-1) + (x-1)^2$

Section 9.7 (page 576)

1. 1.22140
3. 3.32011
5. 0.99619
7. -0.10533
9. 0.42262
11. 1.54306
13. $I(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(2n+1)!} x^{2n+1}$, (all x)
15. $K(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)^2} x^{n+1}, \quad (-1 \le x \le 1)$
17. $M(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(4n+1)} x^{4n+1}, \quad (-1 \le x \le 1)$
19. 0.946
21. 2
23. $-3/25$
25. 0

Section 9.8 (page 580)

1. $\frac{1}{720}(0.2)^7$
3. $\frac{1}{120}(0.5)^5$
5. $\frac{4\sec^2(0.1)\tan^2(0.1)+2\sec^4(0.1)}{4!10^4}$
7. $\frac{24}{120(1.95)^5(20)^5}$
9. $2^x = \sum_{n=0}^{\infty} \frac{(x\ln 2)^n}{n!}$, all x
11. $\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$, all x
13. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $-1 < x < 1$
15. $\frac{x}{2+3x} = \sum_{n=1}^{\infty}(-1)^{n-1}3^{n-1}(\frac{x}{2})^n$, $-\frac{2}{3} < x < \frac{2}{3}$
17. $\sin x = \frac{1}{2}\sum_{n=0}^{\infty} \frac{c_n}{n!}(x - \frac{\pi}{6})^n$, (for all x), where $c_n = (-1)^{n/2}$ if n is even, and $c_n = (-1)^{(n-1)/2}\sqrt{3}$ if n is odd
19. $\ln x = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-1)^n}{n}, \quad 0 < x \le 2$
21. $\frac{1}{x} = -\frac{1}{2}\sum_{n=0}^{\infty}(\frac{x+2}{2})^n$, $-4 < x < 0$

Section 9.9 (page 584)

1. $\sqrt{1+x} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n-3)!!}{2^n n!} x^n, \quad |x| < 1$

3. $\sqrt{4+x} = 2 + \frac{x}{4} + 2\sum_{n=2}^{\infty} (-1)^{n-1} \frac{(2n-3)!!}{2^{3n} n!} x^n, \quad (-4 < x \le 4)$

5. $\sum_{n=0}^{\infty} (n+1)x^n, \quad |x| < 1$

Section 9.10 (page 589)

1. $y = a_0 \left(1 + \sum_{k=1}^{\infty} \frac{(x-1)^{4k}}{4(k!)(3)(7)...(4k-1)}\right) + a_1 \left(x - 1 + \sum_{k=1}^{\infty} \frac{(x-1)^{4k+1}}{4(k!)(5)(9)...(4k+1)}\right)$

3. $y = \sum_{n=0}^{\infty} (-1)^n [\frac{2^n n!}{(2n)!} x^{2n} + \frac{1}{2^{n-1} n!} x^{2n+1}]$

5. $y = 1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \ldots$

7. $y_1 = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{(k!)(2)(5)(8)...(3k-1)}$
   $y_2 = x^{1/3}(1 + \sum_{k=0}^{\infty} \frac{(-1)^k x^k}{(k!)(4)(7)...(3k+1)})$

Review Exercises (page 589)

1. conv. to 0

3. div to ∞

5. $\lim_{n\to\infty} a_n = \sqrt{2}$

7. $4\sqrt{2}/(\sqrt{2}-1)$

9. 2

11. converges

13. converges

15. converges

17. conv. abs.

19. conv. cond.

21. conv. abs. for x in $(-1, 5)$, cond. for $x = -1$, div. elsewhere

23. 1.202

25. $\sum_{n=0}^{\infty} x^n/3^{n+1}, |x| < 3$

27. $1 + \sum_{n=1}^{\infty} (-1)^{n-1} x^{2n}/(ne^n), -\sqrt{e} < x \le \sqrt{e}$

29. $x + \sum_{n=1}^{\infty} (-1)^n 2^{2n-1} x^{2n+1}/(2n)!$, all x

31. $(1/2) + \sum_{n=1}^{\infty} \frac{(-1)^n 1\cdot 4\cdot 7\cdot \ldots \cdot (3n-2) x^n}{2\cdot 24^n n!}, -8 < x \leq 8$

33. $\sum_{n=0}^{\infty} (-1)^n (x-\pi)^n / \pi^{n+1}, \quad 0 < x < 2\pi$

35. $1 + 2x + 3x^2 + \frac{10}{3} x^3$

37. $1 - \frac{1}{2} x^2 + \frac{5}{24} x^4$

39. $\begin{cases} \cos \sqrt{x} & \text{if } x \geq 0 \\ \cosh \sqrt{|x|} & \text{if } x < 0 \end{cases}$

41. $\pi^2/(\pi - 1)^2$

43. $\ln(e/(e-1))$

45. $1/14$

47. 3, 0.49386

Challenging Problems (page 590)

5. (c) 1.645

7. (a) ∞, (c) e^{-x^2}, (d) $f(x) = e^{x^2} \int_0^x e^{-t^2} dt$

CHAPTER 2
Exercises

1. (a) $\frac{s}{s^2-4}, \mathrm{Re}(s) > 2$ (b) $\frac{2}{s^3}, \mathrm{Re}(s) > 0$
   (c) $\frac{3s+1}{s^2}, \mathrm{Re}(s) > 0$ (d) $\frac{1}{(s+1)^2}, \mathrm{Re}(s) > -1$

2. (a) 5 (b) -3 (c) 0 (d) 3 (e) 2
   (f) 0 (g) 0 (h) 0 (i) 2 (j) 3

3. (a) $\frac{5s-3}{s^2}, \mathrm{Re}(s) > 0$
   (b) $\frac{42}{s^4} - \frac{6}{s^2+9}, \mathrm{Re}(s) > 0$
   (c) $\frac{3s-2}{s^2} + \frac{4s}{s^2+4}.\mathrm{Re}(s) > 0$
   (d) $\frac{s}{s^2-9}, \mathrm{Re}(s) > 3$
   (e) $\frac{2}{s^2-4}, \mathrm{Re}(s) > 2$
   (f) $\frac{5}{s+2} + \frac{3}{s} - \frac{2s}{s^2+4}, \mathrm{Re}(s) > 0$
   (g) $\frac{4}{(s+2)^2}, \mathrm{Re}(s) > -2$
   (h) $\frac{4}{s^2+6s+13}, \mathrm{Re}(s) > -3$

(i) $\frac{2}{(s+4)^3}, \mathrm{Re}(s) > -4$

(j) $\frac{36-6s+4s^2-2s^3}{s^4}, \mathrm{Re}(s) > 0$

(k) $\frac{2s+15}{s^2+9}, \mathrm{Re}(s) > 0$

(l) $\frac{s^2-4}{(s^2+4)^2}, \mathrm{Re}(s) > 0$

(m) $\frac{18s^2-54}{(s^2+9)^3}, \mathrm{Re}(s) > 0$

(n) $\frac{2}{s^3} - \frac{3s}{s^2+16}, \mathrm{Re}(s) > 0$

(o) $\frac{2}{(s+2)^3} + \frac{s+1}{s^2+2s+5} + \frac{3}{s}, \mathrm{Re}(s) > 0$

4. (a) $\frac{1}{4}(e^{-3t} - e^{-7t})$ (b) $-e^{-t} + 2e^{3t}$

(c) $\frac{4}{9} - \frac{1}{3}t - \frac{4}{9}e^{-3t}$ (d) $2\cos 2t + 3\sin 2t$

(e) $\frac{1}{64}(4t - \sin 4t)$ (f) $e^{-2t}(\cos t + 6\sin t)$

(g) $\frac{1}{8}(1 - e^{-2t}\cos 2t + 3e^{-2t}\sin 2t)$ (h) $e^t - e^{-t} + 2te^{-t}$

(i) $e^{-t}(\cos 2t + 3\sin 2t)$ (j) $\frac{1}{2}e^t - 3e^{2t} + \frac{11}{2}e^{3t}$

(k) $-2e^{-3t} + 2\cos(\sqrt{2}t) - \frac{1}{\sqrt{2}}\sin(\sqrt{2}t)$ (l) $\frac{1}{5}e^t - \frac{1}{5}e^{-t}(\cos t - 3\sin t)$

(m) $e^{-t}(\cos 2t - \sin 2t)$ (n) $\frac{1}{2}e^{2t} - 2e^{3t} + \frac{3}{2}e^{-4t}$

(o) $-e^t + \frac{3}{2}e^{2t} - \frac{1}{2}e^{-2t}$ (p) $4 - \frac{9}{2}\cos t + \frac{1}{2}\cos 3t$

(q) $9e^{-2t} - e^{-3t/2}[7\cos(\frac{1}{2}\sqrt{3}t - \sqrt{3}\sin(\frac{1}{2}\sqrt{3}t]$

(r) $\frac{1}{9}e^{-t} - \frac{1}{10}e^{-2t} - \frac{1}{90}e^{-t}(\cos 3t + 3\sin 3t)$

5. (a) $x(t) = e^{-2t} + e^{-3t}$

(b) $x(t) = \frac{35}{78}e^{4t/3} - \frac{3}{26}(\cos 2t + \frac{2}{3}\sin 2t)$

(c) $x(t) = \frac{1}{3}(1 - e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t)$

(d) $y(t) = \frac{1}{25}(12^{-t} + 30te^{-t} - 12\cos 2t + 16\sin 2t)$

(e) $x(t) = -\frac{7}{5}e^t + \frac{4}{3}e^{2t} + \frac{1}{15}e^{-4t}$

(f) $x(t) = e^{-2t}(\cos t + \sin t + 3)$

(g) $x(t) = \frac{13}{12}e^t - \frac{1}{3}e^{2t} + \frac{1}{4}e^{-t}(\cos 2t - 3\sin 2t)$

(h) $y(t) = -\frac{2}{3} + t + \frac{2}{3}e^{-t}[\cos(\sqrt{2}t) + \frac{1}{\sqrt{2}}\sin(\sqrt{2}t)]$

(i) $x(t) = (\frac{1}{8} + \frac{3}{4}t)e^{-2t} + \frac{1}{2}t^2e^{-2t} + \frac{3}{8} - \frac{1}{2}t + \frac{1}{4}t^2$

(j) $x(t) = \frac{1}{5} - \frac{1}{5}e^{-2t/3}(\cos\frac{1}{3}t + 2\sin\frac{1}{3}t)$

(k) $x(t) = te^{-4t} - \frac{1}{2}\cos 4t$ (l) $y(t) = e^{-t} + 2te^{-2t/3}$

(m) $x(t) = \frac{5}{4} + \frac{1}{2}t - e^t + \frac{5}{12}e^{2t} - \frac{2}{5}e^{-t}$

(n) $x(t) = \frac{9}{20}e^{-t} - \frac{7}{16}\cos t + \frac{25}{16}\sin t - \frac{1}{80}\cos 3t - \frac{3}{80}\sin 3t$

6. (a) $x(t) = \frac{1}{4}(\frac{15}{4}e^{3t} - \frac{11}{4}e^{t} - e^{-2t}), y(t) = \frac{1}{4}(3e^{3t} - e^{t})$

   (b) $x(t) = 5\sin t + 5\cos t - e^{t} - e^{2t} - 3$
   $y(t) = 2e^{t} - 5\sin t + e^{2t} - 3$

   (c) $x(t) = 3\sin t - 2\cos t + e^{-2t}$
   $y(t) = -\frac{7}{2}\sin t + \frac{9}{2}\cos t - \frac{1}{2}e^{-3t}$

   (d) $x(t) = \frac{3}{2}e^{t/3} - \frac{1}{2}e^{t}, y(t) = -1 + \frac{3}{2}e^{t/3} + \frac{1}{2}e^{t}$

   (e) $x(t) = 2e^{t} + \sin t - 2\cos t$
   $y(t) = \cos t - 2\sin t - 2e^{t}$

   (f) $x(t) = -3 + e^{t} + 3e^{-t/3}$
   $y(t) = t - 1 - \frac{1}{2}e^{t} + \frac{1}{2}e^{-t/3}$

   (g) $x(t) = 2t - e^{t} + e^{-2t}, Y(t) = t - \frac{7}{2} + 3e^{t} + \frac{1}{2}e^{-2t}$

   (h) $x(t) = 3\cos t + \cos(\sqrt{3}t)$
   $y(t) = 3\cos t - \cos(\sqrt{3}t)$

   (i) $x(t) = \cos(\sqrt{\frac{3}{10}}t) + \frac{3}{4}\cos(\sqrt{6}t)$
   $y(t) = \frac{5}{4}\cos(\sqrt{\frac{3}{10}}t) - \frac{1}{4}\cos(\sqrt{6}t)$

   (j) $x(t) = \frac{1}{3}e^{t} + \frac{2}{3}\cos 2t + \frac{1}{3}\sin 2t$
   $y(t) = \frac{2}{3}e^{t} - \frac{2}{3}\cos 2t - \frac{1}{3}\sin 2t$

7. $I_1(s) = \frac{E_1(50+s)s}{(s^2+10^4)(s+100)^2}$

   $I_2(s) = \frac{Es^2}{(s^2+10^4)(s+100)^2}$

   $i_2(t) = E(-\frac{1}{200}e^{-100t} + \frac{1}{2}te^{-100t} + \frac{1}{200}\cos 100t)$

9. $i_1(t) = 20\frac{1}{\sqrt{7}}e^{-t/2}\sin(\frac{1}{2}\sqrt{7}t)$

10. $x_1(t) = -\frac{3}{10}\cos(\sqrt{3}t) - \frac{7}{10}\cos(\sqrt{13}t)$

    $x_2(t) = -\frac{1}{10}\cos(\sqrt{3}t) + \frac{21}{10}\cos(\sqrt{13}t), \quad \sqrt{3}, \quad \sqrt{13}$

13. $f(t) = tH(t) - tH(t-1)$

14. (a) $f(t) = 3t^2 - [3(t-4)^2 + 22(t-4) + 43]H(t-4) -$
    $-[2(t-6) + 4]H(t-6)$
    $F(s) = \frac{6}{s^3} - (\frac{6}{s^3} + \frac{22}{s^2} + \frac{43}{s})e^{-4s} - (\frac{2}{s^3} + \frac{4}{s})e^{-6s}$

    (b) $f(t) = t - 2(t-1)H(t-1) + (t-2)H(t-2)$
    $F(s) = \frac{1}{s^2} - \frac{2}{s^2}e^{-s} + \frac{1}{s^2}e^{-2s}$

15. (a) $\frac{1}{6}(t-5)^3e^{2(t-5)}H(t-5)$

    (b) $\frac{3}{2}[e^{-(t-2)} - e^{-3(t-2)}]H(t-2)$

(c) $[t-\cos(t-1)-\sin(t-1)]H(t-1)$

(d) $\frac{1}{\sqrt{3}}e^{-(t-\pi)/2}\{\sqrt{3}\cos[\frac{1}{2}\sqrt{3}(t-\pi)]+\sin[\frac{1}{2}\sqrt{3}(t-\pi)]\}H(t-\pi)$

(e) $H(t-\frac{4}{5}\pi)\cos 5t$

(f) $[t-\cos(t-1)-\sin(t-1)]H(t-1)$

16. $x(t)=e^{-t}+(t-1)[(1-H(t-1)]$

17.

$$\begin{aligned}x(t) &= 2e^{-t/2}\cos(\frac{1}{2}\sqrt{3}t)+t-1-2H(t-1)\\ &\{t-2+e^{-(t-1)/2}\{\cos[\frac{1}{2}\sqrt{3}(t-1)]\\ &-\frac{1}{\sqrt{3}}\sin[\frac{1}{2}\sqrt{3}(t-1)]\}\}\\ &+H(t-2)\{t-3+e^{-(t-2)/2}\\ &\{\cos[\frac{1}{2}\sqrt{3}(t-2)]-\frac{1}{\sqrt{3}}\sin[\frac{1}{2}\sqrt{3}(t-2)]\}\}\end{aligned}$$

18. $x(t)=e^{-t}+\frac{1}{10}(\sin t-3\cos t+4e^{\pi}e^{-2t}-5e^{\pi/2}e^{-t})H(t-\frac{1}{2}\pi)$

19.

$$\begin{aligned}f(t) &= 3+2(t-4)H(t-4)\\ F(s) &= \frac{3}{s}+\frac{2}{s^2}e^{-4s}\\ x(t) &= 3-2\cos t+2[t-4-\sin(t-4)]H(t-4)\end{aligned}$$

20.

$$\begin{aligned}\theta_0(t) &= \frac{3}{10}(1-e^{-3t}\cos t-3e^{-3t}\sin t)\\ &-\frac{3}{10}[1-e^{3a}e^{-3t}\cos(t-a)\\ &-3e^{3a}e^{-3t}\sin(t-a)]H(t-a)\end{aligned}$$

21.

$$\begin{aligned}\theta_0(t) &= \frac{1}{32}(3-2t-3e^{-4t}-10te^{-4t})\\ &+\frac{1}{32}[2t-3+(2t-1)e^{-4(t-1)}]H(t-1)\end{aligned}$$

23. $\frac{3-3e^{-2s}-6se^{-4s}}{s^2(t-e^{-4s})}$

24. $\frac{K}{T}\frac{1}{s^2} - \frac{K}{s}\frac{e^{-sT}}{1-e^{-sT}}$

25. (a) $2\delta(t) + 9e^{-2t} - 19e^{-3t}$

(b) $\delta(t) - \frac{5}{2}\sin 2t$

(c) $\delta(t) - e^{-t}(2\cos 2t + \frac{1}{2}\sin 2t)$

26. (a) $x(t) = (\frac{1}{6} - \frac{2}{3}e^{-3t} + \frac{1}{2}e^{-4t}) + (e^{-3(t-2)} - e^{-4(t-2)})H(t-2)$

(b) $x(t) = \frac{1}{2}e^{6\pi}e^{3t}H(1-2\pi)\sin 2t$

(c) $x(t) = 5e^{3t} - 4e^{-2t} + (e^{-3(t-3)} - e^{-4(t-3)})H(t-3)$

27. (a) $f'(t) = g'(t) - 43\delta(t-4) - 4\delta(t-6)$

$$g'(t) = \begin{cases} 6t & (0 \le t < 4) \\ 2 & (4 \le t < 6) \\ 0 & (t \ge 6) \end{cases}$$

(b) $$g'(t) = \begin{cases} 1 & (0 \le t < 1) \\ -1 & (1 \le t < 2) \\ 0 & (t \ge 2) \end{cases}$$

(c) $f'(t) = g'(t) + 5\delta(t) - 6\delta(t-2) + 15\delta(t-4)$

$$g'(t) = \begin{cases} 2 & (0 \le t < 2) \\ -3 & (2 \le t < 4) \\ 2t-1 & (t \ge 4) \end{cases}$$

28. $x(t) = -\frac{19}{9}e^{-5t} + \frac{19}{9}e^{-2t} - \frac{4}{3}te^{-2t}$

30. $q(t) = \frac{E}{Ln}e^{-\mu t}\sin nt, \quad n^2 = \frac{1}{LC} - \frac{R^2}{4L^2}, \quad \mu = \frac{R}{2L}$

$i(t) = \frac{E}{Ln}e^{-\mu t}(n\cos nt - \mu\sin nt)$

31. $y(t) = \frac{1}{48EI}[2Mx^4/l + 8W(x-\frac{1}{2}l)^3H(x-\frac{1}{2}l) - 4(M+W)x^3 + (2M+3W)l^2x]$

32.

$$\begin{aligned} y(x) &= \frac{w(x_2^2 - x_1^2)x^2}{4EI} - \frac{w(x_2 - x_1)x^3}{6EI} \\ &\quad + \frac{w}{24EI}[(x-x_1)^4H(x-x_1) - (x-x_2)^4H(x-x_2)] \\ y_{\max} &= wl^4/8EI \end{aligned}$$

33.

$$\begin{aligned} y(x) &= \frac{W}{EI}[\frac{1}{6}x^3 - \frac{1}{6}(x-b)^3H(x-b) - \frac{1}{2}bx^2] \\ &= \begin{cases} -\frac{Wx^2}{6EI}(3b - x) & (0 < x \le b) \\ \\ -\frac{Wb^2}{6EI}(3x - b) & (b < x \le l) \end{cases} \end{aligned}$$

34. (a) $\frac{3s+2}{s^2+2s+5}$
    (b) $s^2+2s+5=0$, order 2
    (c) Poles $-1\pm j2$; zero $-\frac{2}{3}$

35. $\frac{s^2+5s+6}{s^3+5s^2+17s+13}$, $s^3+5s^2+17s+13=0$
    order 3, zeros -3, -2, poles $-1, -2\pm j3$

36. (a) Marginally stable (b) Unstable (c) Stable (d) Stable (e) Unstable

37. (a) Unstable
    (b) Stable
    (c) Marginally stable
    (d) Stable
    (e) Stable

40. $K > \frac{2}{3}$

41. (a) $3e^{-7t}-3e^{-8t}$ (b) $\frac{1}{3}e^{-4t}\sin 3t$
    (c) $\frac{2}{3}(e^{4t}-e^{-2t})$ (d) $\frac{1}{3}e^{2t}\sin 3t$

42. $\frac{s+8}{(s+1)(s+2)(s+4)}$

47. $\frac{2}{7}, \frac{4}{5}$

49. (a) $\frac{1}{54}[(2-e^{-3t}(9t^2+6t+2)]$
    (b) $\frac{1}{125}[e^{-3t}(5t+2)+e^{2t}(5t-2)]$
    (c) $\frac{1}{16}(4t-1+e^{-4t})$

51. $e^{-3t}-e^{-4t}$
    $x(t)=\frac{1}{12}A[1-4e^{-3t}+3e^{-4t}-(1-4e^{-3(t-T)}+3e^{-4(t-T)})H(t-T)]$

52. $e^{-2t}\sin t$, $\frac{1}{5}[1-e^{-2t}(\cos t+2\sin t)]$

2.8 Review exercises

1. (a) $x(t)=\cos t+\sin t-e^{-2t}(\cos t+3\sin t)$
   (b) $x(t)=-3+\frac{13}{7}e^{t}+\frac{15}{7}e^{-2t/5}$

2. (a) $e^{-t}-\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-t}(\cos t+\sin t)$
   (b) $i(t)=2e^{-t}-2e^{-2t}+V[e^{-t}-\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-t}(\cos t+\sin t)]$

3. $x(t)=-t+5\sin t-2\sin 2t$,
   $y(t)=1-2\cos t+\cos 2t$

4. $\frac{1}{5}(\cos t + 2\sin t)$
$e^{-t}[(x_0 - \frac{1}{5})\cos t + (x_1 + x_0 - \frac{3}{5})\sin t]$
$\frac{1}{\sqrt{5}}$, 63.4° lag

6. (a) (i) $\frac{s\cos\phi - \omega\sin\phi}{s^2+\omega^2}$

(ii) $\frac{s\sin\phi + \omega(\cos\phi + \sin\phi)}{s^2+2\omega s+2\omega^2}$

(b) $\frac{1}{20}(\cos 2t + 2\sin 2t) + \frac{1}{20}e^{-2t}(39\cos 2t + 47\sin 2t)$

7. (a) $e^{-2t}(\cos 3t - 2\sin 3t)$

(b) $y(t) = 2 + 2\sin t - 5e^{-2t}$

8. $x(t) = e^{-8t} + \sin t, y(t) = e^{-8t} - \cos t$

9. $q(t) = \frac{1}{500}(5e^{-100t} - 2e^{-200t}) - \frac{1}{500}(3\cos 100t - \sin 100t)$, current leads by approximately 18.5°

10. $x(t) = \frac{29}{20}e^{-t} + \frac{445}{1212}e^{-t/5} + \frac{1}{3}e^{-2t} - \frac{1}{505}(76\cos 2t - 48\sin 2t)$

11. (a) $\theta = \frac{1}{100}(4e^{-4t} + 10te^{-4t} - 4\cos 2t + 3\sin 2t)$

(b) $i_1 = \frac{1}{7}(e^{4t} + 6e^{-3t})$, $i_2 = \frac{1}{7}(e^{-3t} - e^{4t})$

12. $i = \frac{E}{R}[1 - e^{-nt}(\cos nt + \sin nt)]$

13. $i_1 = \frac{E(4-3e^{-Rt/L} - e^{-3Rt/L})}{6R}$, $i_2 \to E/3R$

14. $x_1(t) = \frac{1}{3}[\sin t - 2\sin 2t + \sqrt{3}\sin(\sqrt{3}t)]$

$x_2(t) = \frac{1}{3}[\sin t + \sin 2t - \sqrt{3}\sin(\sqrt{3}t)]$

15. (a) (i) $e^{-t}(\cos 3t + \sin 3t)$
(ii) $e^t - e^{2t} + 2te^t$

(b) $y(t) = \frac{1}{2}e^{-t}(8 + 12t + t^3)$

16. (a) $\frac{5}{2}e^{7t}\sin 2t$

(b) $\frac{n^2 i}{Ks(s^2+2Ks+n^2)}$, $\theta(t) = \frac{i}{K}(1 - e^{-Kt}) - ite^{-Kt}$

17. (a) (ii) $e^{-(t-\alpha)}[\cos 2(t-\alpha) - \frac{1}{2}\sin 2(t-\alpha)]H(t-\alpha)$

(b) $y(t) = \frac{1}{10}[e^{-t}(\cos 2t - \frac{1}{2}\sin 2t) + 2\sin t - \cos t]$
$+ \frac{1}{10}[e^{-(t-\pi)}(\cos 2t - \frac{1}{2}\sin 2t) + \cos t - 2\sin t]H(t-\pi)$

18. $i(t) = \frac{1}{250}[e^{-40t} - 2H(1 - \frac{1}{2}T)e^{-40(t-T/2)} + 2H(t-T)e^{-40(t-T)}$
$- 2H(t - \frac{3}{2}T)e^{-40(t-3T/2)}]$

Yes, since time constant is large compared with T.

19. $e^{-t}\sin t, \frac{1}{2}[1-e^{-t}(\cos t+\sin t)]$

20. $EI\frac{d^4y}{dx^4}=12+12H(x-4)-R\delta(x-4)$,

$y(0)=y'(0)=y(4)=y^{(2)}(5)=y^{(3)}(0)=0$

$$y(x)=\begin{cases}\frac{1}{2}x^4-4.25x^3+9x^2 & (0\le x\le 4)\\ \frac{1}{2}x^4-4.25x^3+9x^2+\frac{1}{2}(x-4)^4-7.75(x-4)^3 & (4\le x\le 5)\end{cases}$$

25.5 kN, 18 kNm

21. (a) $f(t)=H(t-1)-H(t-2)$

$x(t)=H(t-1)(1-e^{-(t-1)})H(t-2)(1-e^{-(t-2)})$

(b) $0, E/R$

23. (a) $t-2+(t+2)e^{-t}$

(b) $y=t+2-2e^t+2te^t, y(t)=\frac{1}{2}t^2+y_1$

24. $EIy=-\frac{2}{9}WIx^2+\frac{10}{81}Wx^3-\frac{W(x-l)^3}{6}H(x-l)$

$EI\frac{d^4y}{dx^4}=-W\delta(x-l)-w[H(x)-H(x-l)]$

25. (a) $x(t)=\frac{1}{6}\{1+e^{3(t-a)/2}[\sqrt{3}\sin(\frac{1}{2}\sqrt{3}t)-\cos(\frac{1}{2}\sqrt{3}t)]H(t-a)\}$

26. (a) No (b) $\frac{1}{s^2+2s+(K-3)}$ (d) $K>3$

27. (a) 4 (b) $\frac{1}{10}$

28. (a) $\frac{K}{s^2+(1+KK_1)s+K}$

(c) $K=12.5, K_1=0.178$ (d) 0.65s, 2.48s, 1.86s

29. (a) $K_2=M_2\omega^2$

30. (b) Unstable (c) $\beta=25\times10^{-5}$, 92dB

(d) -8db, 24°

(e) $K=10^6, \tau_1=10^{-6}, \tau_2=10^{-7}, \tau_3=4\times10^{-8}$

(f) $s^3+36\times10^6s^2+285\times10^{12}s+25\times10^{18}(1+10^7\beta)=0$

CHAPTER 3
Exercises

1. (a) $\frac{4z}{4z-1}, |z|>\frac{1}{4}$ (b) $\frac{z}{z-3}, |z|>3$

(c) $\frac{z}{z+2}, |z|>2$ (d) $\frac{-z}{z-2}, |z|>2$

(e) $3\frac{z}{(z-1)^2}, |z|>1$

2. $e^{-2\omega kT}\leftrightarrow\frac{z}{z-e^{-2\omega T}}$

4. $\frac{1}{z^3}\frac{2z}{2z-1} = \frac{2}{z^2(2z-1)}$

5. (a) $\frac{5z}{5z+1}$ (b) $\frac{z}{z+1}$

6. $\frac{2z}{2z-1}$, $\frac{2z}{(2z-1)^2}$

8. (a) $\{e^{-4kT}\} \leftrightarrow \frac{z}{z-e^{-4T}}$
   (b) $\{\sin kT\} \leftrightarrow \frac{z\sin T}{z^2-2z\cos T+1}$
   (c) $\{\cos 2kT\} \leftrightarrow \frac{z(z-\cos 2T)}{z^2-2z\cos 2T+1}$

11. (a) 1 (b) $(-1)^k$ (c) $(\frac{1}{2})^k$ (d) $\frac{1}{3}(-\frac{1}{3})^k$
    (c) j^k (f) $(-j\sqrt{2})^k$ (g) $0(k=0), 1(k>0)$
    (h) $1(k=0), (-1)^{k+1}(k>0)$

12. (a) $\frac{1}{2}[1-(-2)^k]$ (b) $\frac{1}{7}[3^k - (-\frac{1}{2})^k]$
    (c) $\frac{1}{3} + \frac{1}{6}(-\frac{1}{2})^k$ (d) $\frac{2}{3}(\frac{1}{2})^k + \frac{2}{3}(-1)^{k+1}$
    (e) $\sin\frac{1}{2}k\pi$ (f) $2^k\sin\frac{1}{6}k\pi$
    (g) $\frac{5}{2}k + \frac{1}{4}(1-3^k)$ (h) $k + 2\frac{1}{\sqrt{3}}\cos(\frac{1}{3}k - \frac{3}{2}\pi)$

13. (a) $\{0,1,0,0,0,0,0,2\}$
    (b) $\{1,0,3,0,0,0,0,0,0,-2\}$
    (c) $\{5,0,0,1,3\}$
    (d) $\{0,0,1,1\} + \{(-\frac{1}{3})^k\}$
    (e) $1(k=0), \frac{5}{2}(k=1), \frac{5}{4}(k=2), -\frac{1}{8}(-\frac{1}{2})^{k-3}(k\geq 3)$
    (f) $\begin{cases} 0 & (k=0) \\ 3-2k+2^{k-1} & (k\geq 1)\end{cases}$
    (g) $\begin{cases} 0 & (k=0) \\ 2-2^{k-1} & (k\geq 1)\end{cases}$

14. $y_{k+2} + \frac{1}{2}y_{k+1} = x_k$, $y_{k+2} + \frac{1}{4}y_{k+1} - \frac{1}{5}y_k = x_k$

15. (a) $y_k = k$ (b) $y_k = \frac{3}{10}(9^k) + \frac{17}{10}(-1)^k$
    (c) $2^{k-1}\sin\frac{1}{2}k\pi$ (d) $2(-\frac{1}{2})^k + 3^k$

16. (a) $y_k = \frac{2}{5}(-\frac{1}{2})^k - \frac{9}{10}(\frac{1}{3})^k + \frac{1}{2}$
    (b) $y_k = \frac{7}{2}(3^k) - 6(2^k) + \frac{5}{2}$
    (c) $y_n = \frac{2}{5}(3^n) - \frac{2}{3}(2^n) + \frac{4}{15}(\frac{1}{2})^n$
    (d) $y_n = -2(\sqrt{3})^{n-1}\sin\frac{1}{6}n\pi + 1$

(e) $y_n = -\frac{2}{5}(-\frac{1}{2})^n + \frac{12}{5}(2^n) - 2n - 1$

(f) $y_n = -\frac{1}{2}[2^n + (-2)^n] + 1 - n$

17. (b) 7, 4841

18. $y_k = 2^k - \frac{1}{2}(3^k) + \frac{1}{2}$

19. As $k \to \infty, I_k \to 2G$ as a damped oscillation

21. (a) $\frac{1}{z^2-3z+2}$ (b) $\frac{z-1}{z^2-3z+1}$ (c) $\frac{z+1}{z^3-z^2+2z+1}$

23. (a)$\frac{1}{2}\{(-\frac{1}{4})^k - (-\frac{1}{2})^k\}$ (b) $2(3^k)\sin\frac{1}{6}(k+1)\pi$

(c) $\frac{2}{3}(0.4)^k + \frac{1}{3}(-0.2)^k$ (d) $4^{k+1} + 2^k$

24. $\begin{cases} 0 & (k=0) \\ 2^{k-1}-1 & (k \geq 1) \end{cases}$

$\begin{cases} 0 & (k=0) \\ 2^{k-1} & (k \geq 1) \end{cases}$

25. (a), (b) and (c) are stable; (d) is unstable; (e) is marginally stable.

26. $2 - (\frac{1}{2})^k$

27. $y_n = -4(\frac{1}{2})^n + 2(\frac{1}{3})^n + 2(\frac{2}{3})^n$

29. q form:

$(Aq^2 + Bq + C)y_k = \Delta^2(q^2 + 2q + 1)u_k$

δ form:

$[A\Delta^2\delta^2 + (2\Delta A + \Delta B)\delta + (A + B + C)]y_k]$

$= \Delta^2(4 + 4\Delta\delta + \Delta^2\delta^2)u_k$

$A = 2\Delta^2 + 6\Delta + 4$

$B = 4\Delta^2 - 8$

$C = 2\Delta^2 - 6\Delta + 4$

30. $\frac{1}{s^3+2s^2+2s+1}$

$[(\Delta^4 + 4\Delta^2 + 8\Delta + 8)\delta^3 + (6\Delta^2 + 16\Delta + 16)\delta^2$

$+ (12\Delta + 16)\delta + 8]y_k = (2 + \Gamma\delta)^3 u_k$

32. $\frac{12(z^2-z)}{(12+5\Delta)z^2+(8\Delta-12)z-\Delta}$

$\frac{12\gamma(1+\Delta\gamma)}{\Delta(12+5\Delta)\gamma^2+(8\Delta-12)\gamma+12}$

3.10 Review exercises

1. $3 + 2k$
2. $\frac{1}{6} + \frac{1}{3}(-2)^k - \frac{1}{2}(-1)^k$
7. $\frac{2z}{(z-e)^{3T}} - \frac{z}{z-e^{-2T}}$
8. (a) $\{\frac{1}{a-b}(a^n - b^n)\}$
   (b) (i) $3^{k-1}k$ (ii) $2\frac{1}{\sqrt{3}} \sin \frac{1}{3}k\pi$
9. $\frac{3}{2} - \frac{1}{2}(-1)^k - 2^k$
10. $(-1)^k$
    $\frac{1}{2}A[2 - 2(\frac{1}{2})^k - k(\frac{1}{2})^{k-1}]$

CHAPTER 4
Exercises

1. (a) $f(t) = -\frac{1}{4}\pi - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2n-1)t}{(2n-1)^2} + \sum_{n=1}^{\infty}[\frac{3\sin(2n-1)t}{2n-1} - \frac{\sin 2nt}{2n}]$
   (b) $f(t) = \frac{1}{4}\pi + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2n-1)t}{(2n-1)^2} - \sum_{n=1}^{\infty} \frac{\sin nt}{n}$
   (c) $f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\sin nt}{n}$
   (d) $f(t) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos 2nt}{4n^2-1}$
   (e) $f(t) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos nt}{4n^2-1}$
   (f) $f(t) = \frac{1}{2}\pi - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2n-1)t}{(2n-1)^2}$
   (g) $f(t) = -\frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2n-1)t}{(2n-1)^2} - \sum_{n=1}^{\infty} \frac{\sin 2nt}{n}$
   (h)
$$\begin{aligned} f(t) = &(\frac{1}{2}\pi + \frac{1}{\pi}\sinh \pi) \\ &+ \frac{2}{\pi}\sum_{n=1}^{\infty}[\frac{(-1)^n - 1}{n^2} + \frac{(-1)^n \sinh \pi}{n^2+1}]\cos nt \\ &- \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{n(-1)^n}{n^2+1}\sinh \pi \sin nt \end{aligned}$$
2. $f(t) = \frac{1}{3}\pi^2 + 4\sum_{n=1}^{\infty} \frac{\cos nt}{n^2}$
   Taking $t = \pi$ gives the required result.
3. $q(t) = Q[\frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos(2n-1)t}{(2n-1)^2}]$

4. $f(t) = \frac{5}{\pi} + \frac{5}{2}\sin t - \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nt}{4n^2-1}$

5. Taking $t = 0$ and $t = \pi$ gives the required answers.

6. $f(t) = \frac{1}{4}\pi - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(4n-2)t}{(2n-1)^2}$
   Taking $t = 0$ gives the required series.

7. $f(t) = \frac{3}{2} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n-1)t}{(2n-1)^2}$
   Replacing t by $t - \frac{1}{2}\pi$ gives the following sine series of odd harmonics:

$$f(t - \frac{1}{2}\pi) - \frac{3}{2} = -\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n\sin(2n-1)t}{(2n-1)^2}$$

8. $f(t) = \frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{n\pi t}{l}$

9. $f(t) = \frac{2K}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi t}{l}$

10. $f(t) = \frac{3}{2} + \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\frac{\sin(2n-1)\pi t}{5}$

11. $v(t) = \frac{A}{\pi}(1 + \frac{1}{2}\pi\sin\omega t - 2\sum_{n=1}^{\infty}\frac{\cos 2n\omega t}{4n^2-1})$

12. $f(t) = \frac{1}{3}T^2 + \frac{4T^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi t}{T}$

13. $e(t) = \frac{E}{2}(1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{2\pi nt}{T})$

15. $f(t) = -\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi t$

16. (a) $f(t) = \frac{2}{3} - \frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos 2n\pi t + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\pi t$
   (b) $f(t) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\pi t$
   $+ \frac{2}{\pi}\sum_{n=1}^{\infty}[\frac{1}{2n-1} + \frac{4}{\pi^2(2n-1)^3}]\sin(2n-1)\pi t$

17. $f(t) = \frac{1}{6}\pi^2 - \sum_{n=1}^{\infty}\frac{1}{n^2}\cos 2nt$
   $f(t) = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}\sin(2n-1)t$

18. $f(x) = \frac{8a}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2\pi-1)^2}\sin\frac{(2n-1)\pi x}{l}$

19. $f(x) = \frac{2l}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\frac{2(2n-1)\pi x}{l}$

20. $f(t) = \frac{1}{2}\sin t + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{n(-1)^{n+1}}{4n^2-1}\sin 2nt$

21. $f(x) = \frac{1}{2}A - \frac{4A}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\frac{(2n-1)\pi x}{l}$

22. $T(x) = \frac{8KL^2}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}\sin\frac{(2n-1)\pi x}{L}$

23. $f(t) = \frac{1}{2} + \frac{1}{2}\cos \pi t + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{4n^2-1} \sin 2n\pi t - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)\pi t$

26. (c) $1 + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nt$

29. (a) $\frac{1}{6}\pi^2 + \sum_{n=1}^{\infty} \frac{2}{n^2}(-1)^n \cos nt + \sum_{n=1}^{\infty} \frac{1}{\pi}[-\frac{\pi^2}{n}(-1)^n + \frac{2}{n^3}(-1)^n - \frac{2}{n^3}] \sin nt$

(b)

$$\begin{aligned}
a_n &= 0 \\
b_n &= \frac{4}{n\pi}(\cos n\pi - \cos\frac{1}{2}n\pi) \\
&+ 2(\frac{3\pi}{4n^2}\sin\frac{1}{2}n\pi - \frac{\pi^2}{8n}\cos\frac{1}{2}n\pi \\
&+ \frac{3}{n^3}\cos\frac{1}{2}n\pi - \frac{6}{\pi n^4}\sin\frac{1}{2}n\pi), \\
&\frac{1}{\pi}[(\frac{3}{2}\pi^2 - 16)\sin t + \frac{1}{8}(32 + \pi^3 - 6\pi)\sin 2t \\
&- \frac{1}{3}(\frac{32}{9} + \frac{1}{2}\pi^2)\sin 3t + \ldots]
\end{aligned}$$

(c) $-\frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos(2n-1)\pi t}{(2n-1)^2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\sin(2n-1)t}{(2n-1)}$

(d) $\frac{1}{4} + \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos 2(2n-1)\pi t$

30. $e(t) = 5 + \frac{20}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)100\pi t$

$i_{ss}(t) \simeq 0.008\cos(100\pi t - 1.96) + 0.005\cos(300\pi t - 0.33)$

31. $f(t) = \frac{400}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)t$

$x_{ss}(t) \simeq 0.14\sin(\pi t - 0.1) + 0.379\sin(3\pi t - 2.415) + 0.017\sin(5\pi t - 2.83)$

32. $f(t) = \frac{100}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin 2\pi nt$

$x_{ss}(t) \simeq 0.044\sin(2\pi - 3.13) - 0.0052\sin(4\pi t - 3.14)$

33. $e(t) = \frac{100}{\pi} + 50\sin 50\pi t - \frac{200}{\pi}\sum_{\pi}^{\infty}\sum_{n=1}^{\infty} \frac{\cos 100\pi nt}{4n^2-1}$

$i_{ss}(t) \simeq 0.78\cos(50\pi t + (-0.17)) - 0.01\sin(100\pi t + (-0.48))$

35. $f(t) = \frac{1}{2} + \sum_{n=-\infty, n\neq 0}^{\infty} \frac{j}{2n\pi}[(-1)^n - 1]e^{jnt}$

36. (a) $\frac{3}{4}\pi + \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{2\pi}\{\frac{j\pi}{n} - \frac{1}{n^2}[1 + (-1)^n]\}e^{jnt}$

(b) $\frac{a}{2}\sin\omega t - \sum_{n=-\infty}^{\infty} \frac{a}{2\pi(n^2-1)}[(-1)^n + 1]e^{jn\omega t}$

(c) $\frac{3}{2} + \sum_{n=-\infty, n\neq 0}^{\infty} \frac{j}{2\pi n}[1 - (-1)^n]e^{jnt}$

(d) $\frac{2}{\pi}\sum_{n=-\infty}^{\infty} \frac{1}{1-4n^2} e^{2jnt}$

38. (b) (i) 17.74, (ii) 17.95
(c) 18.14; (i) 2.20% (ii) 1.05%

39. (a) $c_0 = 15, c_n = \frac{30}{jn\pi}(1 - e^{-jn\pi/2})$
$15, \frac{30}{\pi}(1-j), -\frac{30}{\pi}j, -\frac{10}{\pi}(1+j), 0, \frac{6}{\pi}(1-j)$
(b) 15W, 24.30W, 12.16W, 2.70W, 0.97W
(c) 60W
(d) 91.9%

40. 0.19, 0.10, 0.0675

41. (c) $c_0 = 0, c_1 = \frac{3}{2}, c_2 = 0, c_3 = -\frac{7}{8}$

42. (c) $c_0 = \frac{1}{4}, c_1 = \frac{1}{2}, c_2 = \frac{5}{16}, c_3 = 0$

46. (b) $c_1 = 0, c_2 = \sqrt{2\pi}, c_3 = 0, \text{MSE} = 0$

4.9 Review exercises

1. $f(t) = \frac{1}{6}\pi^2 + \sum_{n=1}^{\infty} \frac{2}{n^2}(-1)^n \cos nt + \sum_{n=1}^{\infty}[\frac{\pi}{2n-1} - \frac{4}{\pi(2n-1)^3}] \sin(2n-1)t - \sum_{n=1}^{\infty} \frac{\pi}{2n} \sin 2nt$
Taking $T = \pi$ gives the required sum.

2. $f(t) = \frac{1}{9}\pi + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n^2}\{\cos \frac{1}{3}n\pi - \frac{1}{3}[2 + (-1)^n]\} \cos nt, \quad \frac{2}{9}\pi$

3. (a) $f(t) = \frac{2T}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \sin \frac{2(2n-1)\pi t}{T}$
(b) $-\frac{1}{4}T$;
(c) Taking $t = \frac{1}{4}T$ gives $S = \frac{1}{8}\pi^2$

5. $f(t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n \sin(2n-1)t}{(2n-1)^2}$

8. $f(x) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \sin(2n-1)x$
$f(x) = \frac{1}{4}\pi - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2(2n-1)x}{(2n-1)^2}$

10. (a) $f(t) = \sum_{n=1}^{\infty} \frac{2}{n} \sin nt$
(b) $f(t) = \frac{1}{2}\pi + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)t$

13. (a) $f(t) = \frac{1}{2}\pi - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)t$
(b) $g(t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)t$

15. (a) $v(t) = \frac{10}{\pi} + 5\sin\frac{2\pi t}{T} - \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos\frac{4n\pi t}{T}$
    (b) 25W, 9.01%

16. $g(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)t$
    $f(t) = 1 + g(t)$

18. (b) $\frac{\sin\omega t - \omega\cos\omega t}{1+\omega^2}$ $\quad \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\alpha t - \alpha\cos\alpha t}{(2n-1)(1+\alpha^2)}$
    $\alpha = (4n-2)\pi/T$

19. (c) $T_0 = 1, T_1 = t_1, T_2 = 2t^2 - 1, T_3 = 4t^3 - 3t$
    (d) $\frac{1}{16}T_5 - \frac{5}{8}T_4 + \frac{33}{16}T_3 - \frac{5}{2}T_2 + \frac{95}{5}T_1 - \frac{79}{8}T_0$
    (e) $\frac{33}{4}t^3 - 5t^2 + \frac{91}{16}t - \frac{59}{8}$, $\frac{11}{16}$, $t = -1$

CHAPTER 5
Exercises

1. $\frac{2a}{a^2+\omega^2}$

2. $AT^2 j\omega \text{sinc}^2\frac{\omega T}{2}$

3. $AT\text{sinc}^2\frac{\omega t}{2}$

4. $8K\text{sinc}\,2\omega, 2K\text{sinc}\,\omega, 2K(4\text{sinc}\,2\omega - \text{sinc}\,\omega)$

5. $4\,\text{sinc}\,\omega - 4\,\text{sinc}\,2\omega$

7. $\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$

10. $F_s = \frac{x}{x^2+a^2}, F_c = \frac{x}{x^2+a^2}$

12. $\frac{1}{(1-\omega^2)+3j\omega}$

13. $4\,\text{sinc}\,2\omega - 2\,\text{sinc}\,\omega$

14. $\frac{1}{2}T[\text{sinc}\,\frac{1}{2}(\omega_0-\omega)T + \text{sinc}\,\frac{1}{2}(\omega_0+\omega)T]$

15. $\frac{1}{2}e^{-j\omega T/2}[e^{j\omega_0 T/2}\text{sinc}\,\frac{1}{2}(\omega-\omega_0)T + e^{-j\omega_0 T/2}\text{sinc}\,\frac{1}{2}(\omega+\omega_0)T]$

16. $j[\text{sinc}\,(\omega+2) - \text{sinc}\,(\omega-2)]$

18. $4AT\cos\omega t\,\text{sinc}\,\omega T$

19. High-pass filter

20. $\pi c^{-a|\omega|}$

21. $T[\text{sinc}\,(\omega-\omega_0)T + \text{sinc}\,(\omega+\omega_0)T]$

26. $\frac{1}{2}\pi j[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]-\frac{\omega_0}{\omega_0^2-\omega^2}$

28. $\{2,0,2,0\}$

29. $\{2,0,2,0\}$

5.9 Review exeercises

1. $\frac{\sin\omega}{\omega^2}-\frac{\cos 2\omega}{\omega}$

2. $-\frac{\pi j}{\omega}\operatorname{sinc} 2\omega$

7. (a) $\frac{1}{a-b}(e^{at}-e^{bt})H(t)$
   (b) (i) $te^{2t}H(t)$ (ii) $(t-1+e^{-t})H(t)$

8. (a) $-\sin\omega_0(t+\frac{1}{4}\pi)$ (b) $\cos\omega_0 t$
   (c) $je^{j\omega_0 t}$ (d) $-je^{-j\omega_0 t}$

17. (a) $\frac{a+2\pi s}{a^2+4\pi^2 s^2}$
    (b) $\frac{1}{2\pi s}(\sin 2\pi sT-\cos 2\pi sT+1)$