Lösn. Matematisk Analys i en variabel E, TNIU37, 120831

1) a) $\int_0^1 \frac{2x}{(x^2+1)^2}dx = \left[-\frac{1}{2}\frac{1}{x^2+1}\right]_0^1 = -\frac{1}{2}\left(\frac{1}{5}-\frac{1}{1}\right) = \frac{2}{5}$ b) $\int \ln\sqrt{x}\,dx = \frac{1}{2}\int \ln x\,dx = [PI] =$

$= \frac{1}{2}\left(x\ln x - \int x\cdot\frac{1}{x}dx\right) = \frac{1}{2}(x\ln x - x) + C$ c) $\int \tan x\,dx = \int \frac{1}{\cos x}\sin x\,dx =$

$= -\ln|\cos x| + C$

2) $y' - x^2y = 0$ IF: $e^{\int -x^2dx} = e^{-x^3/3}$ ∴ $\frac{d}{dx}\left(e^{-x^3/3}y\right) = e^{-x^3/3}\cdot 0 = 0 \Rightarrow$

$y = Ce^{x^3/3}$ och $1 = y(1) = Ce^{1/3} \Rightarrow C = e^{-1/3}$ ∴ $y = e^{\frac{1}{3}(x^3-1)}$

3) $y = y_p + y_h$ y_h: kar. ekv.: $r^2 - 2r - 3 = 0 \Rightarrow r = -1, 3 \Rightarrow y_h =$

$= c_1e^{-x} + c_2e^{3x}$ Ansätt $y_p = Cx^m e^{-x} = (m=1) = Cxe^{-x} \Rightarrow y_p' = Ce^{-x} - Cxe^{-x} =$

$= Ce^{-x} - y_p$, $y_p'' = -Ce^{-x} - (Ce^{-x} - Cxe^{-x}) = -2Ce^{-x} + Cxe^{-x}$ ∴ $e^{-x} = y_p'' - 2y_p'$

$-3y_p = \dots = -4Ce^{-x} \Rightarrow C = -\frac{1}{4}$ ∴ $y_p = -\frac{1}{4}xe^{-x}$ ∴ $0 = y(0) = C_1 + C_2$,

$0 = y'(0) = -C_1 + 3C_2 - \frac{1}{4}$ ∴ $C_2 = 1/16$, $C_1 = -1/16$ ∴ $y = -\frac{1}{16}e^{-x} + \frac{1}{16}e^{3x} - \frac{1}{4}xe^{-x} =$

$= -\frac{1}{4}\left(x + \frac{1}{4}\right)e^{-x} + \frac{1}{16}e^{3x}$

4) $$\frac{24\left(1 - \frac{x^2}{2} + \frac{x^4}{4!} + O(x^6)\right) + 12x^2 - 24}{x^2 - x\left(x - \frac{x^3}{3!} + O(x^5)\right)} = \frac{\cancel{x^4}\left(1 + O(x^2)\right)}{\cancel{x^4}\left(\frac{1}{6} + O(x^2)\right)} \rightarrow \frac{1}{1/6} = 6$$

5) a) $(r+2)(r-1) = r^2 + r - 2 \Leftrightarrow y_h = C_1e^{x} + C_2e^{-2x} \Leftrightarrow y'' + y' - 2y = 0$

b) $\left.\begin{matrix} y = y_p + y_h \\ y_p = x^2 \end{matrix}\right\}$ $\boxed{y'' + y' - 2y = 2 + 2x - 2x^2}$

6) $Vol = \int_0^1 \pi f(x)^2 dx = \pi\int_0^1 \frac{x(1-x)}{(x+2)^2(x+1)}dx$. PBU: $\frac{x(1-x)}{(x+2)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \{HP\} =$

$= \frac{-2}{x+1} + \frac{B}{x+2} + \frac{6}{(x+2)^2} = \dots = \frac{(-2+B)x^2 + (-8+3B+6)x + 2B-2}{(x+1)(x+2)^2}$ Ident. av koeff. $\Rightarrow$

∴ $\begin{cases} -2+B = -1 \\ 3B-2 = 1 \\ 2B-2 = 0 \end{cases} \Rightarrow B = 1$ ∴ $Vol = \pi\int_0^1 \frac{-2}{x+1} + \frac{1}{x+2} + \frac{6}{(x+2)^2}dx =$

$= \pi\left[-2\ln|x+1| + \ln|x+2| - \frac{6}{x+2}\right]_0^1 = \dots = \pi\left(\ln\frac{3}{8} + 1\right)$

7) Se lärobok

8) $\lim_{x\to\infty}\int_x^{x+1} \arctan t\,dt = \lim_{x\to\infty}(x+1-x)\arctan\xi$ där $\xi \in [x, x+1]$

$= \lim_{\xi\to\infty}\arctan\xi = \frac{\pi}{2}$