Lösningar Analys i en variabel B1 TMV137 130830

1 a) $\int x(x+1)\,dx = \int x^2+x\,dx = \frac{x^3}{3}+\frac{x^2}{2}+C$ b) $\frac{1}{(x-2)(x-3)} = [PBU] = \frac{A}{x-3}+\frac{B}{x-2} = \cdots = \frac{1}{x-3}-\frac{1}{x-2} \Rightarrow$

$\int_0^1 \frac{1}{(x-2)(x-3)}\,dx = \int_0^1 \frac{1}{x-3}-\frac{1}{x-2}\,dx = \Big[\ln|x-3|-\ln|x-2|\Big]_0^1 = \ln 2-\ln 1-(\ln 3-\ln 2) = \ln\left(\frac{4}{3}\right)$ c) $\int e^{\sqrt{x}}\,dx =$

$= \left[\begin{matrix} t=\sqrt{x} & dx=2t\,dt \\ x=t^2 & \end{matrix}\right] = \int e^t\, 2t\,dt = 2\left(\int te^t\,dt\right) = [PI] = 2\left(te^t-\int 1\cdot e^t\,dt\right) = 2(te^t-e^t+C)$

$= 2(t-1)e^t+C = 2(\sqrt{x}-1)e^{\sqrt{x}}+C$

2) $y'+2y=0,\ y(0)=2$ IF $e^{\int 2\,dx} = e^{2x}$ $\therefore$ $\frac{d}{dx}(e^{2x}y) = 0\cdot e^{2x} = 0 \Rightarrow e^{2x}y = \int \frac{d}{dx}(e^{2x}y)\,dx$

$= \int 0\,dx = C$ $\therefore$ $y = Ce^{-2x}$, $2 = y(0) = Ce^0 = C$ $\therefore$ $y = 2e^{-2x}$

3) $y = y_h+y_p$: y_h: kar. ekv. $0 = r^2+4 \Rightarrow r = \pm 2i \Rightarrow y_h = K_1e^{-2ix}+K_2e^{2ix} =$

$=$ (reell form) $= C_1\cos 2x + C_2\sin 2x$ Ansätt nu $y_p = x^m(A\cos 2x + B\sin 2x)$

$=$ ($m=1$ så att ansatz $y_p \notin y_h$) $= x(A\cos 2x + B\sin 2x) \Rightarrow y_p'' = 4(Bx-A)\sin 2x +$

$+ 4(-Ax+B)\cos 2x$ Insättning i ekv. och identifiering av koeff. $\Rightarrow$

$4(Bx-A)\sin 2x + 4(-Ax+B)\cos 2x + 4x(A\cos(2x)+B\sin(2x)) = (8Bx-4A)\sin 2x + 4B\cos 2x =$

$= \sin(2x) \Rightarrow B=0,\ -4A=1$ $\therefore$ $y = y_p+y_h = \left(-\frac{x}{4}\cos 2x + C_1\right)\cos 2x + C_2\sin 2x$

4) a) Kar. ekv. $(r-1)(r+2) = r^2+r-2 \Rightarrow y''+y'-2y=0$ har lösningen

enligt 3) $y = y_p+y_h$ lösn ~~[illegible]~~ $y''+y'-2y = y_p''+y_p'-2y_p =$

$= 2+2x-2x^2$ där $y_p = x^2$

5) $\sin t = t-\frac{t^3}{3!}+O(t^5)$ $\arctan t = t-\frac{t^3}{3}+O(t^5)$ $\therefore$ $\frac{3\sin x-\sin 3x}{3\arctan x-\arctan 3x} =$

$$= \frac{3\left(x-\frac{x^3}{3!}+O(x^5)\right)-\left(3x-\frac{(3x)^3}{3!}+O(x^5)\right)}{3\left(x-\frac{x^3}{3}+O(x^5)\right)-\left(3x-\frac{(3x)^3}{3}+O(x^5)\right)} = \frac{-\frac{x^3}{2}+\frac{3^2x^3}{2}+O(x^5)}{-x^3+3^2x^3+O(x^5)} = \frac{\cancel{x^3}(4+O(x^2))}{\cancel{x^3}(8+O(x^2))} \to \frac{4+0}{8+0} = \frac{1}{2}$$

6) $y(t) =$ mängden järn i behållaren vid tiden $t \Rightarrow y' = ky-a \Rightarrow y'-ky = -a$

IF: $e^{\int -k\,dt} = e^{-kt} \Rightarrow e^{-kt}y = \int \frac{d}{dt}(e^{-kt}y)\,dt = \int -a e^{-kt}\,dt = -a\frac{e^{-kt}}{-k}+C$

$\Rightarrow y = \frac{a}{k}+Ce^{kt}$, $y_0 = y(0) = \frac{a}{k}+Ce^0 \Rightarrow C = y_0-\frac{a}{k}$ $\therefore$ $y = \frac{a}{k}+\left(y_0-\frac{a}{k}\right)e^{kt}$

$\therefore$ Välj a så att $y_0-\frac{a}{k}=0 \Leftrightarrow a = ky_0 = 0{,}4y_0$

7) $I = \int\sqrt{x^2+1}\,dx = \int\frac{x^2+1}{\sqrt{x^2+1}}\,dx = \int\frac{1}{\sqrt{x^2+1}}\,dx + \int\frac{x^2}{\sqrt{x^2+1}}\,dx = \ln(x+\sqrt{x^2+1}) + \int\frac{x}{\sqrt{x^2+1}}\cdot x\,dx$

$= [PI] = \ln(x+\sqrt{x^2+1}) + \sqrt{x^2+1}\cdot x - \underbrace{\int\sqrt{x^2+1}\cdot 1\,dx}_{I}$ $\therefore$ $2I = \ln(x+\sqrt{x^2+1}) + x\sqrt{x^2+1}+C$

så $\int\sqrt{x^2+1}\,dx = I = \frac{1}{2}\ln(x+\sqrt{x^2+1}) + \frac{1}{2}x\sqrt{x^2+1} + C$

8) Se kursboken.