Lösningar Analys i en variabel, B1, TMV137, 140425

1 a) Första ordningens linjär ODE b) Nej, pga termen xyy' c) $y = x\sin x$

$y' = \sin x + x\cos x$, $y'' = 2\cos x - x\sin x$ så ekv. $yy'' = y(2\cos x - y)$

har bl. a. lösning $y = x\sin x$ (och som vi ju direkt ser $y \equiv 0$)!

2) a) IF: $e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = e^{\ln x^2} = x^2$ så $\frac{d}{dx}(x^2 y) = x^4 \Rightarrow x^2 y = \int \frac{d}{dx}(x^2y)dx =$

$= \int x^4 dx = \frac{x^5}{5} + C \Rightarrow y = \frac{x^3}{5} + Cx^{-2}$ b) $y = y_p + y_h$ Kar. ekv. $0 = r^2+2r+2 =$

$(r+1)^2+1 \Rightarrow r = -1 \pm i \Rightarrow y_h = C_1 e^{(-1-i)x} + C_2 e^{(-1+i)x} = e^{-x}(A\cos x + B\sin x)$

Ansätt $y_p = x^m(ax^2+bx+c) = (m=0) = ax^2+bx+c \Rightarrow y_p' = 2ax+b$, $y_p'' = 2a$

Insättning $\Rightarrow y_p'' + 2y_p' + 2y_p = 2a + 4ax + 2b + 2(ax^2+bx+c) = x^2-1 \Rightarrow a = \frac{1}{2}, b = -1, c = 0$

så $y = y_p + y_h = \frac{1}{2}x^2 - x + e^{-x}(A\cos x + B\sin x)$

3) För tangenten $y = y(x_0) + \ldots$ till lösningen $y = y(x)$ i punkten $x = x_0$ gäller

k = linjens riktningskoefficient $= y'(x_0) = \frac{y - y(x_0)}{x - x_0} = \frac{y - y_0}{x - x_0}$ Tag $x = x_1 = x_0 + h$, h = steglängd

så $y_1 \approx y = y_0 + (x_1 - x_0)y'(x_0) = y_0 + hy'(x_0) = y_0 + hf(x_0, y(x_0)) = y_0 + hf(x_0, y_0)$

Pss $y_{i+1} = y_i + hf(x_i, y_i)$, $x_{i+1} = x_i + h$

4) a) $\lim_{x\to 0} \frac{(e^{-x^2}-1)\sin x}{x\ln(1+x^2)} = \lim_{x\to 0}\frac{\sin x}{x} \lim_{x\to 0}\frac{e^{-x^2}-1}{\ln(1+x^2)} = 1\cdot \lim_{x\to 0}\frac{1+(-x^2)+O(x^4)-1}{x^2+O(x^4)} =$

$= \lim_{x\to 0}\frac{x^2(-1+O(x^2))}{x^2(1+O(x^2))} = \frac{-1+0}{1+0} = -1$

b) $\int_1^e \frac{\ln x \arctan(\ln x)}{x}dx = [t = \ln x] = \int_0^1 t\arctan t\, dt = [PI] = \left[\frac{t^2}{2}\arctan t\right]_0^1 - \frac{1}{2}\int_0^1 t^2\frac{1}{1+t^2}dt =$

$= \frac{1}{2}\frac{\pi}{4} - 0 - \frac{1}{2}\int_0^1 \frac{t^2+1-1}{t^2+1}dt = \frac{\pi}{8} - \frac{1}{2}\left(\int_0^1 1 - \frac{1}{t^2+1}dt\right) = \frac{\pi}{8} - \frac{1}{2}\left([t]_0^1 - [\arctan t]_0^1\right) = \frac{\pi}{8} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right) = \frac{\pi}{4} - \frac{1}{2}$

5) $\int \frac{4x-5}{x^3-7x^2+16x-10}dx = \int \frac{4x-5}{(x-1)(x^2-6x+10)}dx = \int \frac{4x-5}{(x-1)((x-3)^2+1)}dx = [PBU] = \int \frac{-1/5}{x-1} + \frac{\frac{1}{5}x+3}{x^2-6x+10}dx$

$= -\frac{1}{5}\ln|x-1| + \frac{1}{5}\int \frac{x+15}{x^2-6x+10}dx = -\frac{1}{5}\ln|x-1| + \frac{1}{10}\left(\int \frac{2x-6}{x^2-6x+10}dx + \int \frac{36}{(x-3)^2+1}dx\right) = -\frac{1}{5}\ln|x-1| + \frac{1}{10}\ln(x^2-6x+10) + \frac{18}{5}\arctan(x-3) + C$

6) Omkrets $= 4 = 4a + 4b$ $\quad Vol_x = \pi\int_0^a b^2dx + \pi\int_a^{2a}(2b)^2dx$ $\quad Vol_y = \int_0^a 2\pi x b\,dx + \int_a^{2a} 2\pi x\, 2b\,dx = 18\pi a^2 b$

$V_x = 5\pi a b^2$ så $V = V_x V_y = 90\pi^2(1-b)^3b^3 = 90\pi^2(b-b^2)^3 \Rightarrow V' = 90\pi^2\cdot 3(b-b^2)^2(1-2b)$

b	0		$1/2$		1
V'	0	$+$	0	$-$	0
V	0	$\nearrow$		$\searrow$	0

$\Rightarrow V_{max} = V(\frac{1}{2}) = 90\pi^2/64$ antas då $b = 1/2$ (som också ses genom symmetrin)

7) $y = y_p + y_h$ Kar. ekv. $r^2+1 = 0 \Rightarrow r = \pm i \Rightarrow y_h = C_1\cos x + C_2\sin x$ Då $\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$ så ser

$y_p = y_{p_1} + y_{p_2} = \frac{1}{2} + x(A\cos x + B\sin x)$ så $y = y_{p_1} + y_{p_2} + y_h = \frac{1}{2} + \frac{x}{4}\sin x + C_1\cos x + C_2\sin x$

(Se uppg. 5 tenta 131218 för utförligare lösn.)

8) Se kursliteraturen.