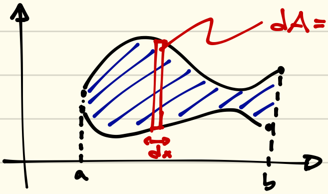
Repetition 2

Måndag:

- Areor av plana områden

$$\int_a^b f(x)\,dx = \text{"arean under grafen till } f(x) \text{ räknad med tecken"}$$

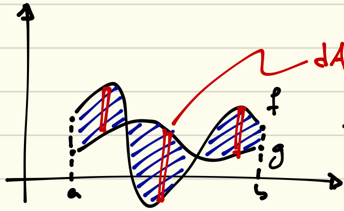
(i)



$dA = (f(x)-g(x))\,dx$

$$\Rightarrow A = \int_a^b (f(x)-g(x))\,dx$$

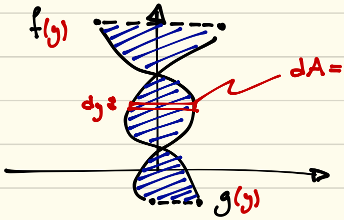
(ii)



$dA = |f(x)-g(x)|\,dx$

$$\Rightarrow A = \int_a^b |f(x)-g(x)|\,dx$$

(iii)



$dA = |f(y)-g(y)|\,dy$

$$\Rightarrow A = \int_a^b |f(y)-g(y)|\,dy$$

- Partiell integration

$$\int_a^b f(x)\cdot g(x)\,dx = \left\{ \begin{matrix} F(x) & \text{prim. till } f(x) \\ g'(x) & \text{deriv. till } g(x) \end{matrix} \right\} =$$

$$= \Big[F(x)\cdot g(x)\Big]_a^b - \int_a^b F(x)\cdot g'(x)\,dx$$

$$\int \ln(x)\,dx = \int 1\cdot\ln(x)\,dx = x\cdot\ln(x) - \int x\cdot\frac{1}{x}\,dx = x\ln(x) - x + C.$$

Onsdag:

- Partialbråksuppdelning

$$\int \frac{p(x)}{q(x)}\,dx = ?$$

(i) om grad(p) > grad(q); polynomdiv! $\rightsquigarrow \frac{p(x)}{q(x)} = v(x) + \frac{r(x)}{q(x)}$

(ii) annars/därefter; faktorisera $q(x)$!

$$q(x) = (x-a_1)^{m_1}\cdot(x-a_2)^{m_2}\cdot\ldots\cdot(x-a_n)^{m_n}\cdot(x^2+b_1x+c_1)^{n_1}\cdot\ldots\cdot(x^2+b_kx+c_k)^{n_k}$$

$$\Rightarrow \frac{r(x)}{(x-a_1)\cdot\ldots\cdot(x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \ldots + \frac{A_n}{x-a_n}$$

Ex: $\frac{x^2+2x-1}{(x+1)(x-1)(x-2)} = \frac{A_1}{x+1} + \frac{A_2}{x-1} + \frac{A_3}{x-2}$

$$\Rightarrow A_1 = -1/3,\ A_2 = -1,\ A_3 = 7/3$$

- Inversa substitutioner

$$\int f(x)\,dx = \cancel{\left\{\begin{matrix} u = h(x) \\ du = h'(x)\,dx \end{matrix}\right\}} \left\{\begin{matrix} x = h(u) \\ dx = h'(u)\,du \end{matrix}\right\} = \ldots$$

$$\int \sqrt{a^2-x^2}\,dx = \left\{\begin{matrix} x = a\cdot\sin\theta \\ dx = a\cdot\cos\theta\,d\theta \end{matrix}\right\} = \ldots = \int a^2\cos^2\theta\,d\theta$$

Fredag:

- Inversa substitutioner (forts.)

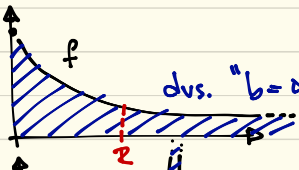
$$\int \text{"rationell funktion i } \sin\theta \text{ och } \cos\theta\text{"}\, d\theta$$

Sätt $x = \tan\frac{\theta}{2}$ $\rightsquigarrow$ $\begin{cases} \cos\theta = \frac{1-x^2}{1+x^2} \\ \sin\theta = \frac{2x}{1+x^2} \\ d\theta = \frac{2}{1+x^2}dx \end{cases}$

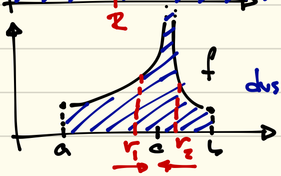
Ex: $\displaystyle\int \frac{\sin\theta + \cos^2\theta}{1+\sin\theta}\,d\theta = \int \frac{\frac{2x}{1+x^2} + \left(\frac{1-x^2}{1+x^2}\right)^2}{1 + \frac{2x}{1+x^2}} \cdot \frac{2}{1+x^2}\,dx = \int \frac{p(x)}{q(x)}\,dx$

- Generaliserade integraler

Beräkna $\displaystyle\int_a^b f(x)\,dx$



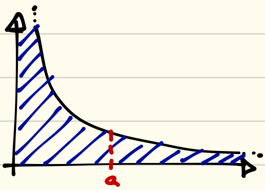
dvs. "$b = \infty$"



dvs. "$f(c) = \infty$"

Sats:

$f(x) = x^{-p}$



$$\int_0^a x^{-p}dx = \begin{cases} \frac{a^{1-p}}{1-p}, & p<1 \\ \infty, & p \geq 1 \end{cases}$$

$$\int_a^\infty x^{-p}dx = \begin{cases} \infty, & p \leq 1 \quad \text{divergent} \\ \frac{a^{1-p}}{p-1}, & p>1 \quad \text{konvergent} \end{cases}$$