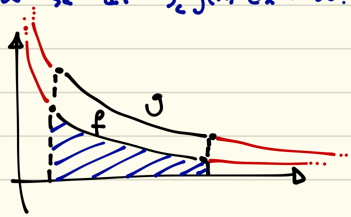
Repetition 3

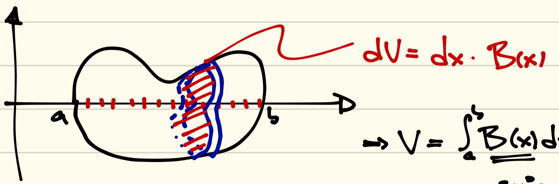
Måndag:

- Jämförelsekriteriet

<u>Sats:</u> Låt $-\infty \leq a < b \leq \infty$ och antag att f och g är kont. på (a,b) och att $0 \leq f(x) \leq g(x)$. Om $\int_a^b g(x)\,dx$ är <u>konvergent</u> så är även $\int_a^b f(x)\,dx$ det, och $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.
Om $\int_a^b f(x)\,dx = \infty$ så är $\int_a^b g(x)\,dx = \infty$.



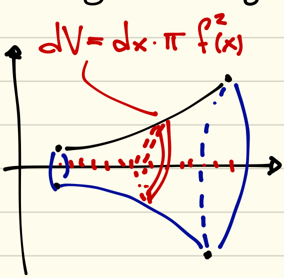
- Volymer och kurvlängder

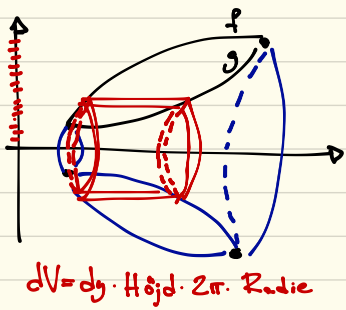


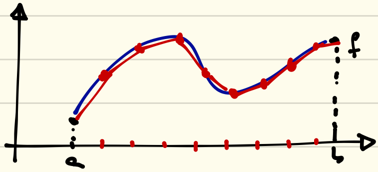
$\Rightarrow V = \int_a^b \underline{B(x)}\,dx$

svår att hitta!

Om volymen rot.sym. så kan man hitta $B(x)$!





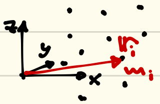


$S_n \approx S$

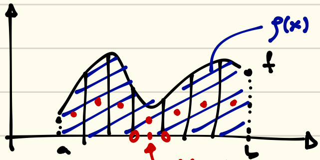
$$S_n = \sum_{i=1}^{n} \sqrt{\Delta x^2 + \Delta y_i^2} = \sum_{i=1}^{n} \sqrt{1+\left(\frac{\Delta y_i}{\Delta x}\right)^2} \cdot \Delta x \xrightarrow{n\to\infty} \int_a^b \sqrt{1+(f'(x))^2}\, dx$$

Onsdag:

- Tyngdpunkter



$$\bar{r} = (\bar{r}_x, \bar{r}_y, \bar{r}_z) = \frac{\sum_{i=1}^{n} \bar{r}_i \cdot m_i}{\sum_{i=1}^{n} m_i}$$



$\longrightarrow \bar{r} = ?$

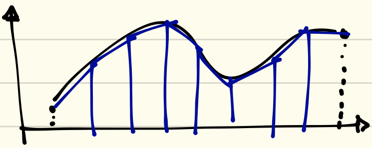
$$\frac{x_{i-1}+x_i}{2} = a_i$$

$$dm = \rho(a_i)\cdot f(a_i)\cdot dx \rightsquigarrow m = \int_a^b \rho(x) f(x)\, dx$$

$$\sum_{i=1}^{n} a_i \cdot \rho(a_i)\cdot f(a_i)\, dx \rightsquigarrow \int_a^b x\cdot \rho(x) f(x)\, dx = M_{x=0}$$

$$\sum_{i=1}^{n} \frac{1}{2} f(a_i)\cdot \rho(a_i) f(a_i)\, dx \rightsquigarrow \frac{1}{2}\int_a^b f^2(x)\cdot \rho(x)\, dx = M_{y=0}$$

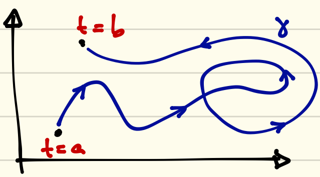
- Numeriska metoder



$$\int_a^b f(x)\, dx = ?$$

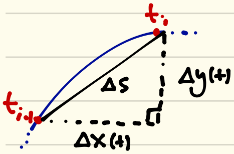
Fredag:

- Kurvor i planet



$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}, \quad t \in I$$

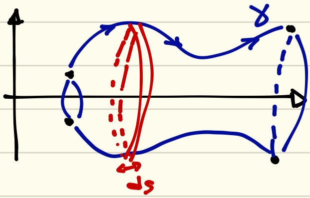
$s = ?$



$$\Delta s = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2} \cdot \Delta t$$

$$\rightsquigarrow ds = \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

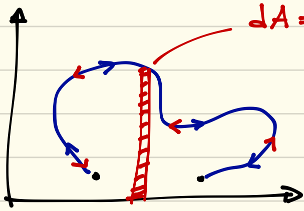
$$\Rightarrow \quad s = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$



$A = ?$

$dA = ds \cdot 2\pi |g(t)|$

$$\Rightarrow A = \int_a^b 2\pi |g(t)| \cdot \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$



$$dA = g(t)\, dx = g(t) \cdot \frac{dx}{dt} \cdot dt =$$
$$= g(t)\, f'(t)\, dt$$

$$A = \int_a^b g(t) f'(t)\, dt \begin{cases} > 0 \\ < 0 \end{cases}$$