

1 Lösningsförslag till dugga 2, TMV138/TMV181, 20161124

1.1 Dugga 2a)

- (a) Ja
(b) Nej
(c) Nej
(d) Nej
- (a) $\sin x - x \cos x + C$
(b) $\frac{\ln 10}{2}$

3.

$$\int_{-1}^2 \frac{x+1}{\sqrt{x+2}} dx = \left\{ \text{V.S. } \sqrt{x+2} = t \Leftrightarrow x = t^2 - 2 \Rightarrow dx = 2tdt - 1 \leq x \leq 2, 1 \leq t \leq 2 \right\} =$$

$$\int_0^2 \frac{t^2-1}{t} 2tdt = 2 \left[\frac{t^3}{3} - t \right]_1^2 = 2 \left(\frac{8-1}{3} - 2 + 1 \right) = \frac{8}{3}$$

eller

$$\begin{aligned} \int_{-1}^2 \frac{x+2-1}{\sqrt{x+2}} dx &= \int_{-1}^2 \left((x+2)^{1/2} - (x+2)^{-1/2} \right) dx = \left[\frac{2}{3}(x+2)^{3/2} - 2(x+2)^{1/2} \right]_{-1}^2 \\ &= \frac{2}{3}(4^{3/2} - 1^{3/2}) - 2(4^{1/2} - 1^{1/2}) = \frac{8}{3}. \end{aligned}$$

1.2 Dugga 2b)

- (a) Ja
(b) Ja
(c) Ja
(d) Ja
- (a) $\sin x - x \cos x + C$
(b) $\sqrt{2} - 1$

3.

$$\int_0^3 \frac{x}{\sqrt{x+1}} dx = \left\{ \text{V.S.: } \begin{array}{l} \sqrt{x+1} = t \Leftrightarrow x = t^2 - 1 \Rightarrow \\ dx = 2tdt \quad \begin{array}{l} 0 \leq x \leq 3 \\ 1 \leq t \leq 2 \end{array} \end{array} \right\} = \int_1^2 \frac{t^2-1}{t} 2tdt = 2 \left[\frac{t^3}{3} - t \right]_1^2 =$$

$$2 \left[\frac{8}{3} - 2 - \left(\frac{1}{3} - 1 \right) \right] = \frac{8}{3}$$

eller

$$\begin{aligned} \int_0^3 \frac{x}{\sqrt{x+1}} dx &= \int_0^3 \frac{x+1-1}{\sqrt{x+1}} dx = \int_0^3 \left((x+1)^{1/2} - (x+1)^{-1/2} \right) dx = \left[\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} \right]_0^3 \\ &= \frac{2}{3}(4^{3/2} - 0^{3/2}) - (4^{1/2} - 0^{1/2}) = \frac{2}{3}(8 - 2) = \frac{8}{3}. \end{aligned}$$