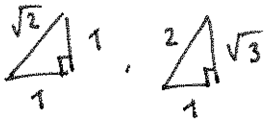
Envariabelsanalys Z & TD, ht2017, Räkneövning 2.1

5.5.9 Beräkna $\displaystyle\int_{-\pi/4}^{-\pi/6} \cos(x)\,dx$

<u>Lösn.:</u> $\displaystyle\int_{-\pi/4}^{-\pi/6} \cos(x)\,dx = \Big[\sin(x)\Big]_{-\frac{\pi}{4}}^{-\frac{\pi}{6}} = \sin\left(-\frac{\pi}{6}\right) - \sin\left(-\frac{\pi}{4}\right) =$

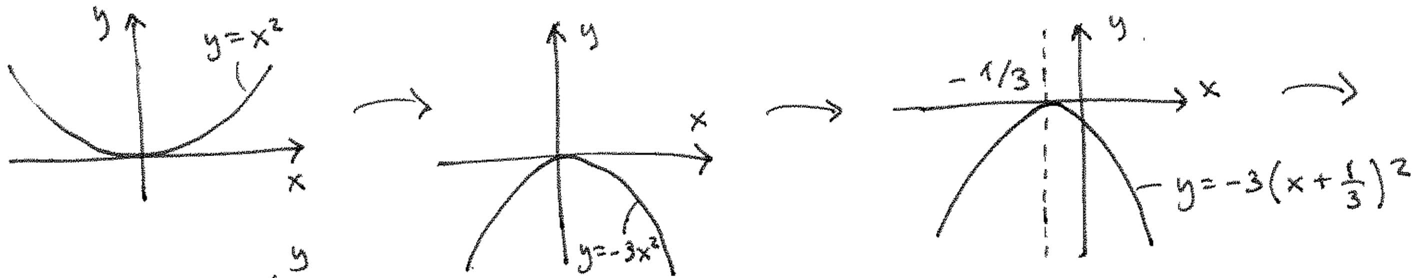
$= \sin\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right) = \Big\{$  $\Big\} =$

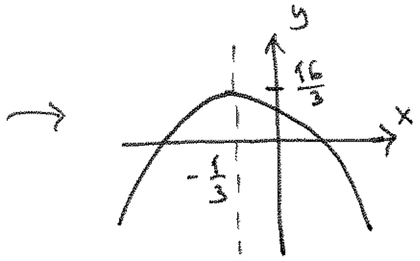
$= \dfrac{1}{\sqrt{2}} - \dfrac{1}{2} = \dfrac{2-\sqrt{2}}{2\sqrt{2}}$

5.5.25 Beräkna arean av området som begränsas av kurvorna $y = 5 - 2x - 3x^2$, $y = 0$, $x = -1$, $x = 1$.

<u>Lösn.:</u> $y = 5 - 2x - 3x^2 = -3\left(x^2 + \frac{2}{3}x - \frac{5}{3}\right) =$

$= -3\left(\left(x+\frac{1}{3}\right)^2 - \frac{1}{9} - \frac{5\cdot 3}{3\cdot 3}\right) = -3\left(x+\frac{1}{3}\right)^2 + \frac{16}{3}$





$y = 0 \iff x^2 + \frac{2}{3}x - \frac{5}{3} = 0 \Rightarrow$

$\Rightarrow x = -\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{5\cdot 3}{3\cdot 3}} = -\frac{1}{3} \pm \frac{4}{3} \Rightarrow \begin{array}{l} x_1 = 1 \\ x_2 = -\frac{5}{3} \end{array}$

$\therefore$ Area $= \displaystyle\int_{-1}^{1} (5 - 2x - 3x^2)\,dx = \Big[5x - x^2 - x^3\Big]_{-1}^{1} =$

$= 5 - 1 - 1 - (-5 - \cancel{1} + \cancel{1}) = 8$ a.e.

5.5.44 Beräkna $\displaystyle\frac{d}{d\theta}\int_{\sin\theta}^{\cos\theta} \frac{1}{1-x^2}\,dx$

Lösn.: Låt $S(t) = \int_a^t \frac{1}{1-x^2}\,dx$ där $a \in \mathbb{R}$ konstant

Då gäller att:

(i) $S'(x) \overset{\text{☺}}{=} \frac{1}{1-x^2}$

(ii) $\int_{\sin\theta}^{\cos\theta} \frac{1}{1-x^2}\,dx = \int_{\sin\theta}^{a} \frac{1}{1-x^2}\,dx + \int_{a}^{\cos\theta} \frac{1}{1-x^2}\,dx =$

$= -\int_a^{\sin\theta} \frac{1}{1-x^2}\,dx + \int_a^{\cos\theta} \frac{1}{1-x^2}\,dx = -S(\sin\theta) + S(\cos\theta)$

$\Rightarrow \frac{d}{d\theta} \int_{\sin\theta}^{\cos\theta} \frac{1}{1-x^2}\,dx \overset{(ii)}{=} \frac{d}{d\theta}\left(-S(\sin\theta) + S(\cos\theta)\right) =$

$= -S'(\sin\theta)\cdot\cos\theta + S'(\cos\theta)\cdot(-\sin\theta) \overset{(i)}{=}$

$\overset{(i)}{=} -\frac{1}{1-\sin^2\theta}\cdot\cos\theta - \frac{1}{1-\cos^2\theta}\cdot\sin\theta =$

$= -\frac{1}{\cos^2\theta}\cos\theta - \frac{1}{\sin^2\theta}\cdot\sin\theta = -\frac{1}{\cos\theta} - \frac{1}{\sin\theta}$

5.6.6. Beräkna $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

Lösn.: $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \left\{ \begin{array}{l} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}}\,dx \end{array} \right\} = \int \sin(t)\, 2dt =$

$= -2\cos(t) = -2\cos(\sqrt{x}) + C$

5.6.19 Beräkna $\int \tan(x)\ln(\cos(x))\,dx$

Lösn.: $\int \tan(x)\ln(\cos(x))\,dx = \int \frac{\sin(x)}{\cos(x)} \ln(\cos(x))\,dx =$

$$= \left\{ \begin{array}{l} t = \ln(\cos(x)) \\ dt = \frac{1}{\cos(x)} \cdot (-\sin(x))\, dx \end{array} \right\} = -\int t\, dt = -\frac{1}{2}t^2 =$$

$$= -\frac{1}{2}\left(\ln(\cos(x))\right)^2 + C$$

5.6.34 Beräkna $\int \frac{\sin^3(\ln(x))\cos^3(\ln(x))}{x}\, dx$

Lösn.: $\int \frac{\sin^3(\ln(x))\cos^3(\ln(x))}{x}\, dx = \left\{ \begin{array}{l} t \doteq \ln(x) \\ dt = \frac{1}{x} dx \end{array} \right\} =$

$$= \int \sin^3(t)\cos^3(t)\, dt = \int \sin^3(t)\left(1 - \sin^2(t)\right)\cos(t)\, dt =$$

$$= \left\{ \begin{array}{l} u = \sin(t) \\ du = \cos(t)\, dt \end{array} \right\} = \int u^3(1 - u^2)\, du = \int (u^3 - u^5)\, du =$$

$$= \frac{1}{4}u^4 - \frac{1}{6}u^6 = \frac{1}{4}\sin^4(t) - \frac{1}{6}\sin^6(t) =$$

$$= \frac{1}{4}\sin^4(\ln(x)) - \frac{1}{6}\sin^6(\ln(x)) + C$$